0.) Introduction and history

- Description of classical mechanics in terms of <u>Eqs. of motion</u>

→ Newton's second law:

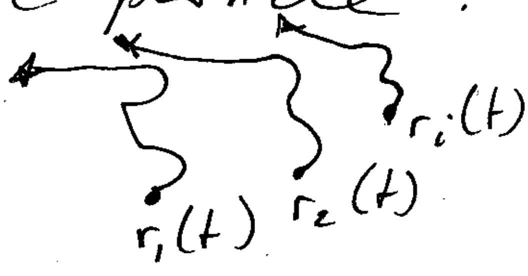
$$\underline{F}(\underline{r}, \underline{p}) = \frac{d}{dt}\,\underline{p} = \dot{\underline{p}}$$

and $\dot{\underline{r}} = \frac{\underline{p}}{m}$

- Kinematics: <u>description</u> of motion of particles

Dynamics: <u>origin</u> — " —

- Variables:
  - in Euclidean space: 3D (one for each particle:



- in configuration space: 3N-D (N: number of particles)

obviously: $\underline{R}(t) = \{\underline{r}_1(t), \underline{r}_2(t) \ldots\}$

⟹ are equivalent

- phase space : $2n$-D
(n: number of degrees of freedom,
e.g. 3 for mass point in 3D
2 for 2 independent points
moving on circle
etc ...

"2" : for r and p
coordinate momentum)

→ space of choice for most numerical systems (only first-order derivatives) and quantum mechanics

- History:

Ancient Greeks: very good at math + statics, bad in dynamics/motion
e.g. Pythagoras: concept that <u>complex</u> phenomena can be understood in terms of simpler ones.

- Copernicus/Kepler/Galileo (~1500+)

  Sun at center
  law of planetary motions
  " " gravity

- Descartes (~1500)

  analytic geometry

- Newton: first <u>full theory</u> of

  forces
  behavior of mass points
  absolute time and space

  ↑ recognizes that in practice
  both is relative, but absolute
  <u>should</u> exist
  (from that later: concept of
  symmetries, relativity)

I) Survey of the elementary principles

- Review -

A.) Single particles

velocity: $\underline{v} = \frac{d}{dt}\underline{r} = \dot{\underline{r}}$

momentum: $\underline{p} = m\,\underline{v}$

force: $\underline{F} = \dot{\underline{p}}$

(for constant force:

$$\underline{F} = m\ddot{\underline{r}}\;)$$

⇒ 1st conservation theorem:

$$\boxed{\text{If no force is applied, the linear momentum } \underline{p} \text{ is conserved.}}$$

angular momentum: $\underline{L} = \underline{r} \times \underline{p}$

torque: $\underline{N} = \underline{r} \times \underline{F}$

or (with

$$\frac{d}{dt}(\underline{r}\times m\underline{v}) = \cancel{\underline{v}\times m\underline{v}}^{0} + \underline{r}\times\frac{d}{dt}m\underline{v}$$

$$\underline{N} = \dot{\underline{L}}$$

$\Rightarrow$ 2nd conservation ~~theorem~~:

> If total torque is zero, angular momentum is conserved.

(Assume m constant)

work: $$W_{12} = \int_1^2 \underline{F}\cdot\underline{ds}$$

$$= m\int_{t_1}^{t_2}\frac{d\underline{v}}{dt}\cdot\underline{v}\,dt = \frac{m}{2}\int_{t_1}^{t_2}\frac{d}{dt}v^2\,dt$$

kinetic energy: $$= T_2 - T_1 = \frac{m}{2}v_2^2 - \frac{m}{2}v_1^2$$

$\Rightarrow$ if W_{12} is the same for __any__ path from 1 to 2 (or for

$$\oint\underline{F}\cdot\underline{ds} = 0 \quad)$$

the force $\underline{F}$ is __conservative__.

In this case: $\exists V(\underline{r})$: $\boxed{\underline{F} = -\underline{\nabla} V}$

potential

$\Rightarrow W_{12} = V_2 - V_1$

$\Rightarrow \boxed{T_1 + V_1 = T_2 + V_2}$ energy conservation

$\Rightarrow$ 3rd conservation theorem:

$\boxed{\text{if only conservative forces are present, energy } T+V \text{ is conserved.}}$

symmetries: Galilean group:

if space is isotropic: $\underline{r}' = A\,\underline{r}$ ①

" homogenous: $\underline{r}' = \underline{r} + \Delta\underline{r}$ ②

if motion is relative $\underline{r}' = \underline{r} + \underline{v}\,\Delta t$ ③

if time is homogenous: $t' = t + \Delta t$ ④

for all four symmetries: physics does not change between $\underline{r}', t'$ and $\underline{r}, t$

$\Rightarrow$ conservation laws