Name: _____

Date: _____

Collaborators: _____

(If applicable, the collaborators submit their individually written assignments together)

| Question: | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points: | 5 | 25 | 35 | 5 | 10 | 80 |
| Score: | | | | | | |

**Instructor/grader comments:**

**Krylov subspaces**

1. (5 points) Modify the script `krylovunstable.m` that we used in class such that it uses the following 100x100 upper triangular matrix

```
nn = 100;
lambda = 0.01*(1:nn);
A = triu(ones(nn),1) + diag(lambda);
```

and the seed vector

```
u = ones(nn, 1);
```

Run the code for $ndim = 30$ iterations and observe the (lack of) convergence. Notice that the convergence is much worse than the convergence in the original script.

Store the modified script as **hw03krylovunstable.m**

2. (25 points) For the matrix $A$ from the previous problem build the Krylov matrix $K_{30}$. Calculate and plot 2-norm based condition numbers $\kappa(K_m)$ for $m = 1,\ldots,20$.

Provide axes labels, a grid, and a title for your graph.

Place the commands `clear`, `clf` at the top of your script.

Place the code for this problem into the script named **hw03p2.m**.

In your README file explain how the results of you calculation can explain the failure of naive Krylov approach that you observed in Problem 1.

**GMRES**

3. Consider the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

   (a) (0 points) (Pen and paper problem) Find the exact solution by inspection.

   (b) (10 points) (Pen and paper problem) Find the orthonormal basis of the Krylov subspace, $Q_4$. Use $\mathbf{b}$ as the seed vector.

   (c) (10 points) (Pen and paper problem) Find $H_3$

(d) (15 points) Write matlab script that calculates $Q_k$ and $H_k$ (do not take into account the special structure of the matrix $A$) and finds the GMRES approximate solution $x_k = Q_k z_k$ for $k = 1, \ldots, 4$. Print residues for all solutions. Store the code in the file **hw03p3.m**

4. (5 points) I watched the video Who invented the great numerical algorithms? which is a part of Homework 3 assignment.

Sign and date here: _____

**Gitlab**

5. (10 points) Create a gitlab project called **hw03** (name it exactly as shown). Upload **all** Matlab files that are needed to run your code.

   Create README.md file and write in there your answer to Problem 2.

   Scan your answers/solutions of Problems 3 and 4, combine all scans into a single pdf file (call it **hw03.pdf**) and upload it to gitlab. **Do not** upload other types of files (e.g. no graphics files).

   Share the project with the instructor and the grader and grant them **Reporter** privileges.