Question:	1	2	3	4	5	Total
Points:	15	25	25	25	10	100

Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form.

The method of deflation

1. (15 points) Recall that for the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$A \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

and

$$A\left(\begin{array}{c}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{array}\right) = -\left(\begin{array}{c}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{array}\right).$$

Using the information above construct a matrix with the same eigenvectors as *A* but with the eigenvalues -1 and 0.

Similarity transformation

2. (25 points) (Pen and paper problem.) Recall that for the matrix

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

where
$$i = \sqrt{-1}$$
,

$$A\left(\frac{\frac{1}{\sqrt{2}}}{\frac{i}{\sqrt{2}}}\right) = \left(\frac{\frac{1}{\sqrt{2}}}{\frac{i}{\sqrt{2}}}\right)$$

and

$$A\left(\frac{\frac{i}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right) = -\left(\frac{\frac{i}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right).$$

Define a function of a matrix using function's Taylor series. Find $e^{(At)}$ where t is a real parameter. Show all your work.

Krylov methods

3. (Pen and paper problem.) Consider the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

- (a) (5 points) Find the exact solution by inspection.
- (b) (5 points) Find the Krylov basis K_3
- (c) (5 points) Find the orthonormal basis of the Krylov subspace Q_3
- (d) (10 points) Find H_3

Matlab programming

4. (25 points) Consider the following model for blurring images:

$$B_{ij} = \begin{cases} \frac{1}{16} & |i-j| = 2, \\ \frac{3}{16} & |i-j| = 1, \\ \frac{1}{2} & i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Load a matlab-provided demo image, blur it, and use gmres matlab function to restore the original image. Use the tolerance 10^{-6} .

Check whether the restored and the original image are the same. If not, list possible reasons why they are different.

Git and Gitlab

- 5. (10 points) Upload the code you wrote for this exam to UConn gitlab server:
 - 1. Use gitlab web interface to create a new project called **midterm1-sample**. (The name must be exactly as shown.)
 - 2. Use gitlab web interface to add *README.md* file and edit it to add the relevant content.
 - 3. Use gitlab web interface to **upload** your matlab code to your project. (Do not copy and paste.)
 - 4. Use gitlab web interface to grant the access to your project (with the permission of the *reporter*) to the instructor.