

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Collaborators: \_\_\_\_\_

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	5	25	25	5	60
Score:					

Instructor/grader comments:

**Krylov subspaces**

1. (5 points) Modify the script `krylovunstable.m` that we used in class such that it uses the following 100x100 upper triangular matrix

```
nn = 100;
lambda = 0.01*(1:nn);
A = triu(ones(nn),1) + diag(lambda);
```

and the seed vector

```
u = ones(nn, 1);
```

Run the code for  $np = 20$  iterations and observe the (lack of) convergence. Store the modified script as **hw03krylovunstable.m**

2. (25 points) For the matrix from the previous problem build the Krylov matrix  $K_{20}$ . Calculate and plot 2-norm based condition numbers  $\kappa(K_m)$  for  $m = 1, \dots, 20$ .

Place the code for this problem into the script named **hw03p2.m**.

In your README file explain how the results of you calculation can explain the failure of naive Krylov approach that you observed in Problem 1.

**GMRES**

3. Consider the linear system  $A\mathbf{x} = \mathbf{b}$ , where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

- (a) (5 points) (Pen and paper problem) Find the exact solution by inspection.
- (b) (5 points) (Pen and paper problem) Find the orthonormal basis of the Krylov subspace,  $Q_4$
- (c) (5 points) (Pen and paper problem) Find  $H_4$
- (d) (10 points) Write matlab script that finds the GMRES approximate solution  $x_m$  for  $m = 1, \dots, 4$ . Evaluate residues for all solutions. Use your matrices  $Q_4$  and  $H_4$ . Store the code in the file **hw03p3.m**

**Gitlab**

4. (5 points) Create a gitlab project called **hw03** (name it exactly as shown). Upload **all** files that are required to run your matlab code and create your README.md file. Share the project with the instructor and the grader.