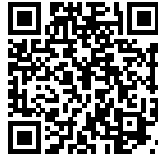


CONVERGENCE RATE OF POWER ITERATIONS

http://www.phys.uconn.edu/~rozman/Courses/m3511_19s/



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$$A \mathbf{v}_i = \lambda_i \mathbf{v}_i, \quad |\lambda_1| \geq |\lambda_2| \geq \dots \quad (1)$$

$$\mathbf{x}^{(k)} = A \mathbf{x}^{(k-1)}, \quad \frac{\mathbf{x}^{(k)}}{|\mathbf{x}^{(k)}|} \rightarrow \mathbf{x}^{(k)} \quad (2)$$

$$\mathbf{x}^{(k)} = \mathbf{v}_1 + \gamma^k \beta \mathbf{v}_2 + \dots \quad (3)$$

$$\gamma = \frac{\lambda_2}{\lambda_1} \quad (4)$$

$$|\mathbf{x}^{(k)}| = \sqrt{(\mathbf{v}_1 + \gamma^k \beta \mathbf{v}_2)^t (\mathbf{v}_1 + \gamma^k \beta \mathbf{v}_2)} = \sqrt{1 + \gamma^{2k} \beta^2} \approx 1 + \frac{1}{2} \gamma^{2k} \beta^2 \approx 1 \quad (5)$$

$$\lambda_1^{(k)} = (\mathbf{x}^{(k)})^t A \mathbf{x}^{(k)} = (\mathbf{v}_1 + \gamma^k \beta \mathbf{v}_2)^t A (\mathbf{v}_1 + \gamma^k \beta \mathbf{v}_2) \quad (6)$$

$$= \lambda_1 + \gamma^{2k} \beta^2 \lambda_2 = \lambda_1 (1 + \gamma^{2k+1} \beta^2) \quad (7)$$

$$\delta_k = \lambda_1^{(k)} - \lambda_1^{(k+1)} = \gamma^{2k+1} \beta^2 (1 - \gamma^2) \quad (8)$$

$$\delta_k \sim \gamma^{2k} = e^{2k \ln \gamma} \quad (9)$$