CONVERGENCE RATE OF POWER ITERATIONS

http://www.phys.uconn.edu/~rozman/Courses/m3511_19s/

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$$A\,\mathbf{v}_i = \lambda_i \mathbf{v}_i, \quad |\lambda_1| \geq |\lambda_2| \geq \dots \tag{1}$$

$$\mathbf{x}^{(k)} = A\,\mathbf{x}^{(k-1)}, \qquad \frac{\mathbf{x}^{(k)}}{\left|\mathbf{x}^{(k)}\right|} \to \mathbf{x}^{(k)} \tag{2}$$

$$\mathbf{x}^{(k)} = \mathbf{v}_1 + \gamma^k \beta\,\mathbf{v}_2 + \dots \tag{3}$$

$$\gamma = \frac{\lambda_2}{\lambda_1} \tag{4}$$

$$\left|\mathbf{x}^{(k)}\right| = \sqrt{\left(\mathbf{v}_1 + \gamma^k \beta\,\mathbf{v}_2\right)^t \left(\mathbf{v}_1 + \gamma^k \beta\,\mathbf{v}_2\right)} = \sqrt{1 + \gamma^{2k}\beta^2} \approx 1 + \frac{1}{2}\gamma^{2k}\beta^2 \approx 1 \tag{5}$$

$$\lambda_1^{(k)} = \left(\mathbf{x}^{(k)}\right)^t A\,\mathbf{x}^{(k)} = \left(\mathbf{v}_1 + \gamma^k \beta\,\mathbf{v}_2\right)^t A\left(\mathbf{v}_1 + \gamma^k \beta\,\mathbf{v}_2\right) \tag{6}$$

$$= \lambda_1 + \gamma^{2k}\beta^2\,\lambda_2 = \lambda_1\left(1 + \gamma^{2k+1}\beta^2\right) \tag{7}$$

$$\delta_k = \lambda_1^{(k)} - \lambda_1^{(k+1)} = \gamma^{2k+1}\beta^2\left(1 - \gamma^2\right) \tag{8}$$

$$\delta_k \sim \gamma^{2k} = e^{2k\ln\gamma} \tag{9}$$