

Question:	1	2	3	4	5	Total
Points:	20	30	30	10	10	100

## I. Pen and paper problems

### Eigenvalues and eigenvectors

1. Given the matrix  $A$  and its not normalized eigenvector,  $\mathbf{v}$ ,

$$A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix},$$

- (a) (10 points) find the eigenvalue,  $\lambda_1$ , that corresponds to the eigenvector  $\mathbf{v}$ . Use Rayleigh quotient.
- (b) (10 points) construct a matrix,  $B$ , that has the same eigenvectors as  $A$  with the eigenvalue  $\lambda_1$  replaced with zero, the other eigenvalues of  $B$  being the same as eigenvalues of  $A$ .

### Gram-Schmidt Orthogonalization

2. (30 points) Perform the Gram-Schmidt Process on the columns of matrix  $A$  and give the associated QR factorization.

$$A = \begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$$

Verify the orthogonality of matrix  $Q$  and the correctness of factorization.

## II. Matlab programming

### Linear least squares

3. (30 points) Use linear least squares approximation and QR decomposition to fit the noisy data returned by the function `m2samplenoisydata()`

```
1 function [x, y] = m2samplenoisydata()
```

using the expression

$$y(x) = a_1 \frac{1}{x} + a_2 \cos(x) + a_3 e^{-x^2/4}.$$

Write the function

```
1 function a = m2qrfit(x, y)
2 % M2QRFIT qr least squares fit of (x,y) data.
3 % Return the coefficients of the best fit
4 %  $p(x) = a(1)/x + a(2)*\cos(x) + a(3)*\exp(-x^2/4)$ 
```

and call it from your main script. Use the QR factorization code that was developed in class.

On the same graph plot your noisy data and you least squares fit. Provide the axis labels, title, legend and the grid for your graph.

Place your code to the matlab script **m2p3.m**

The file m2samplenoisydata.m is available for download on the course website.

4. (10 points) Write matlab script that verifies that the matrix  $B$  that you constricted in Problem 1 has indeed the required eigenvalues and eigenvectors. Use matlab's `eigs` `eig` function.

### Git and Gitlab

5. (10 points) Upload the code you wrote for this exam to UConn gitlab server:
1. Use gitlab web interface to create a new project called **midtermr2-practice**. (The name must be exactly as shown.)
  2. Use gitlab web interface to add *README.md* file and edit it to add the relevant content.
  3. Use gitlab web interface to **upload** your matlab code to your project. (Do not copy and paste.)
  4. Use gitlab web interface to grant the access to your project (with the permission of the *reporter*) to the instructor.