

Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together)

New: if the submitted code is not runnable (e.g. because the project is not shared with the instructor, or because not all needed files have been uploaded, or because the code has not been tested in the first place, or for any other reason that has been under complete student's control, the homework grade is decreased by 10%.

Question:	1	2	3	4	Total
Points:	20	20	5	5	50
Score:					

Instructor/grader comments:

Boundary value problem for PDEs

1. (20 points) Find the numerical solution to the following elliptic partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)e^{xy}, \quad 0 < x < 1, \quad 0 < y < 1,$$

with the following boundary conditions:

$$u(0, y) = 1, \quad u(1, y) = e^y, \quad 0 \leq y \leq 1; \quad u(x, 0) = 1, \quad u(x, 1) = e^x, \quad 0 \leq x \leq 1.$$

Use the Jacobi method for solving the linear system. Plot your solution.

Compare the result to the exact solution $u_e(x, y) = e^{xy}$. Chose the mesh size and the number of iterations so that the maximal error of your numerical solution, $\text{abs}(u(x, y) - u_e(x, y)) \leq 10^{-6}$. Plot the error of your solution.

2. (20 points) Find the numerical solution to the following elliptic partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{x}{y} + \frac{y}{x}, \quad 1 < x < 2, \quad 1 < y < 2,$$

with the following boundary conditions:

$$u(1, y) = y \log y, \quad u(2, y) = 2y \log(2y), \quad 1 \leq y \leq 2;$$

$$u(x, 1) = x \log x, \quad u(x, 2) = x \log(4x^2), \quad 1 \leq x \leq 2.$$

Use the Jacobi method for solving the linear system. Plot your solution.

Compare the result to the exact solution $u_e(x, y) = xy \log(xy)$. Chose a mesh size and plot the maximal absolute error of your numerical solution, $v = \text{abs}(u(x, y) - u_e(x, y))$; $\text{err} = \max(v(:))$, vs. the number of iterations. Chose the maximal number of iterations so that the absolute error is less than 10^{-4} .

Gitlab

3. (5 points) Create a gitlab project called **hw10** (name it exactly as shown). Upload **all** required matlab code and create your readme file. Share the project with the instructor.
4. (5 points) Clearly describe your code design and your observations in the project's readme file.