Question:	1	2	3	4	5	Total
Points:	15	25	10	40	10	100

Floating point numbers

1. Floating point numbers typically represented in computers in the following binary form:

$$\pm \left(1 + \frac{b_1}{2} + \frac{b_2}{2^2} + \ldots + \frac{b_d}{2^d}\right) \times 2^E$$

- (a) (5 points) What is the (approximate) value of machine epsilon for a microprocessor that uses d = 8? Briefly explain.
- (b) (5 points) For the same microprocessor, how many floating point numbers x, such that $4 \le x < 5$ are there? Briefly explain.

(c) (5 points) For the same microprocessor, assuming that the smallest value of *E* is -16, what is (approximately) the smallest positive floating point number? Briefly explain.

Systems of linear equations

2. The chemical equation

$$x_1[Ca(OH)_2] + x_2[HNO_3] \to x_3[Ca(NO_3)_2] + 2[H_2O]$$

indicates that x_1 molecules of calcium hydroxide $Ca(OH)_2$ combine with x_2 molecules of nitric acid HNO_3 to yield x_3 molecules of calcium nitrate $Ca(NO_3)_2$ and 2 molecules of water H_2O .

Since atoms are not destroyed or created in chemical reactions, the balance of oxygen atoms requires that

$$2x_1 + 3x_2 = 6x_3 + 2.$$

The balance of hydrogen atoms requires that

$$2x_1 + x_2 = 4.$$

The balance for nitrogen atoms requires that

$$x_2 = 2x_3$$

(a) (5 points) Rewrite the balance equations above in matrix form Ax = b:

(b) (5 points) Verify that the following two matrices are indeed the results of LU-factorization of A:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{pmatrix}, \qquad U = \begin{pmatrix} 2 & 3 & -6 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{pmatrix}.$$

(c) (5 points) Use *L* and *U* to calculate the determinant of matrix *A*. Write you calculations below:

(d) (5 points) Use the forward substitution to solve the equation Ly = b. Write you calculations below:

(e) (5 points) Use the backward substitution to solve the equation Ux = y. Verify by direct substitution that *x* is the solution of Ax = b. Write you calculations below:

3. (10 points) You wrote your own function to solve a system of linear equations. It takes about 10 seconds (on a slow computer) to solve the system of 100 equations with 100 unknowns. Estimate how long it would take to solve a system of 200 linear equations with 200 unknowns if your code implements LU-factorization method to solve the equations. Present your answer and explain your reasoning in the gitlab's README.md file.

Matlab

4. (40 points) TBA

Git and Gitlab

- 5. (10 points) Upload all the code you wrote/used for this exam:
 - 1. Create a new gitlab project called **midterm1-sample** (the name must be exactly as shown)
 - 2. Add README.md file to your project and edit it to add some meaningful content
 - 3. Upload your matlab code to your project
 - 4. Grant the access to your project (with the permission of the Reporter) to the instructor.