Name: _____

Date: _____

Section: _____

Collaborators: _____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	40	40	5	85
Score:				

Instructor/grader comments:

Roots of nonlinear equations

1. Apply Newton's method to find the single point of intersection of the spheres with center $1,0,1)$ and radius $\sqrt{8}$, center $(0,2,2)$ and radius $\sqrt{2}$, and center $(0,3,3)$ and radius $\sqrt{2}$.

   Hint: although a generic intersection of three spheres in in three-dimensional space is two points, it can be a single point.

   Hint: In Cartesian coordinates the equation of a sphere of radius R and the center at x_c, y_c, z_c is as following:

   $$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = R^2.$$

   (a) (10 points) Write down the system of equations that you need solve:

   (b) (10 points) Calculate the Jacobian.

   (c) (10 points) Write matlab function `hw08p1spheres(x)` that accept the vector of position in three-dimensional space and returns the values of residues and the Jacobian.

   (d) (10 points) Write matlab script **hw08p1.m** that uses the code we developed in class and finds the intersection point. Use the usual rules for writing scripts. In the README file (a) specify your initial approximation and the number of the iterations; (b) does the iterations converge quadratically? Explain.

Numerical derivatives

2. (a) (20 points) Determine the weights in the following one-sided formula for the first derivative of a function $f(x)$:

   $$\frac{\mathrm{d}f}{\mathrm{d}x} = a\,f(x) + b\,f(x+h) + c\,f(x+2h) + d\,f(x+3h).$$

   Use the Fornberg method as described in the class handout. Show your results in your README.md file. Use a computer algebra system to calculate the needed Taylor expansion coefficients.

   (b) (20 points) Conduct numerical experiment to determine the leading error term of your formula $\delta \sim h^\alpha$: write a script, **hw08p2.m**, that for $h = \frac{1}{2^2}, \frac{1}{2^{5/2}}, \frac{1}{2^3}, \frac{1}{2^{7/2}}, \ldots, \frac{1}{2^8}$ calculates the absolute value of the error, $|\delta|$, for the first derivative of $f(x) = \sin(x)$ at $x = 1$ when using your formula. Plot, in double logarithmic axes, the graph of the error $|\delta|$ vs. h. In addition, as a guide, plot the graphs of $y(h) = h^k$, for $k = 1, 2, 3, 4$. Analyze your figure and by visual inspection determine the constant α. Describe your results in gitlab's README.md file.

Follow the course requirements for writing matlab scripts and producing graphs.

Gitlab

3. (5 points) Create a gitlab project called **hw08** (name it exactly as shown). Upload **all** matlab files that are required to run your code. Scan your calculations and results for Problem 1 (a) and (b), and Problem 2(a). Convert the scan to a single pdf file and upload it to gitlab. Share the project with the instructor and the TA and grant them **Reporter** privileges.