Question:	1	2	3	4	5	6	7	8	Total
Points:	5	5	10	10	25	10	25	10	100

Answer the questions in the spaces provided. If you need more space for your answer, continue on the back of the page. Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form.

1. (5 points) Find l_1, l_2 and l_∞ norms of the following vector: $x = (6, 2, -9)$

2. (5 points) Find the maximal values of l_1 and l_∞ norms of the following vector: $x = \left(\sin^2 k, \cos^2 k, 1\right)$ for arbitrary real k

3. (10 points) Find l_∞ and l_1 norms of the matrix.

$$\begin{bmatrix} \sin^2 x & \cos^2 x & 0 & 1 \\ \cos^2 x & -\sin^2 x & 1 & 0 \\ 0 & 0 & -\sin^2 x & -\cos^2 x \\ -1 & -1 & \cos^2 x & \sin^2 x \end{bmatrix}$$

4. (10 points) You are solving a nonlinear equation $f(x) = 0$ using **bisection** and starting from the initial interval $[1, 2]$. Estimate the number of function evaluations that is required to achieve the error less than 2^{-24}.

5. A hypothetical algorithm for solution of nonlinear equation requires three function evaluation per iteration and achieves the **cubic** convergence rate (per iteration).

   (a) (10 points) During the testing you noticed that after the three initial iterations the error of the solution was 10^{-2}. What are the expected order of magnitudes of the errors for the fourth and the fifth iterations? Explain.

   (b) (15 points) What is the convergence rate of the algorithm measured in function evaluations? Is the algorithm slower or faster than the Newton's method (as measured per function evaluation)?

6. (10 points) Find the **leading** in n term (assume $n \gg 1$) of the following sum:

$$S(n) = \sum_{i=1}^{n} i^2 \log(i)$$

Show all your calculations in the space below.

Matlab

7. The Airy function (or Airy function of the first kind), $Ai(x)$, is a special function named after the British astronomer George Airy (1801–1892). The function Ai(x) is a solutions to the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - xy = 0.$$

The Airy function has many important applications: it provides uniform semiclassical approximations near a turning point of Schrödinger's equation; it describes the intensity near an optical directional caustic, such as that of the rainbow; it describes the pattern, due to diffraction and interference, produced by a point source of light.

Matlab's function `airy(x)` returns the value $Ai(x)$; the function `airy(1,x)` returns the value of the first derivative of Airy function.

Write a matlab script, **m2p6.m**, that produces the following:

(a) (5 points) Plot a graph of Airy function for $-10 \le x \le 0$. Provide the grid, a title, and axis labels. Use at least 200 points to produce the graph.

(b) (15 points) Using the matlab code that was developed in class for the bisection, the secant, and the Newton's methods, find the locations of two closest to zero negative roots of Airy function. Add your solutions as symbols to the graph.

(c) (5 points) Compare the performance of the three methods in terms of the number of function evaluations. Describe your observations in gitlab's README file.

Git and Gitlab

8. (10 points) Upload all the code you wrote/used for this exam:
    1. Use gitlab web interface to create a new project called **midterm2-sample** (the name is case sensitive, must be exactly as shown)
    2. Use gitlab web interface to create *README.md* file
    3. Use gitlab web interface to upload your matlab code to your project
    4. Use gitlab web interface to grant the access to your project (with the permission of the *reporter*) to the instructor.