Name: _____

Date: _____

Section: _____

Collaborators: _____

(Collaborators submit their individually written assignments together)

This is the last homework assignment of the semester. There will be no late submissions and resubmissions for corrections.

Question:	1	2	3	4	Total
Points:	30	20	30	5	85
Score:					

Instructor/grader comments:

Numerical integration

Conduct numerical experiments to determine the leading error terms of the following integration rules:

- Simpson's approximation

$$\int_a^b f(x)\,\mathrm{d}x = \frac{h}{3}[f(a)+4f(a+h)+2f(a+2h)+4f(a+3h)+\ldots+4f(b-h)+f(b)]+\mathrm{O}\left(h^{\beta}\right),$$

- three eight's formula

$$\int_a^b f(x)\,\mathrm{d}x = \frac{3h}{8}[f(a)+3f(a+h)+3f(a+2h)+2f(a+3h)+\ldots+3f(b-h)+f(b)]+\mathrm{O}\left(h^{\gamma}\right),$$

Programming hints:

- To divide a segment $[a,b]$ into k subsegments of equal size, use matlab function `linspace(a,b,k+1)`.
- To sum all elements of an array `v`, use `sum(v)`. To sum every third element starting with the 4th element, use `sum(v(4:3:end))`.

1. (a) (15 points) Write a matlab function, `hw10p1simp(h, f)`, that accept the value of the discretization step h and the array of function values $f(a)$, $f(a+h)$, $f(a+2h)$, $\ldots$ $f(b)$ and return the approximation value of the integral using Simpson's rule.

   (b) (15 points) Write a matlab script, **hw10p1**, that for $h=(b-a)/2^n$, where $n=1,2,\ldots 8$ calculates the absolute value of the error for the integral of $f(x)=sin(x)$ between $a=1$ and $b=2$. Plot the graph of the error vs. h in double logarithmic axes. In addition, as guides, plot the graphs of $y(h)=h^k$, for $k=1,2,3,4$. Analyze your figure and by visual inspection determine the constant β. Describe your results in gitlab's README.md file.

2. (a) (10 points) Write a matlab function, `hw10p2te(h, f)`, that accept the value of the discretization step h and the array of function values $f(a)$, $f(a+h)$, $f(a+2h)$, $\ldots$ $f(b)$ and return the approximation value of the integral using three eight's rule.

   (b) (10 points) Write a matlab script, **hw10p2**, that for $h=(b-a)/(3*2^{n-1})$, where $n=1,2,\ldots 8$ calculates the absolute value of the error for the integral of $f(x)=sin(x)$

between $a = 1$ and $b = 2$. Plot the graph of the error vs. h in double logarithmic axes. In addition, as guides, plot the graphs of $y(h) = h^k$, for $k = 1, 2, 3, 4$. Analyze your figure and by visual inspection determine the constant γ. Describe your results in gitlab's README.md file.

Improved Euler method for IVP

Conduct numerical experiments to determine the leading error terms in the following two-stage method for the solution of the first order IVP:

$$u' = f(t, u), \quad a \le t \le b, \quad u(a) = u_0.$$

In the first stage the method takes an Euler half-step to time $t_i + h/2$:

$$k_1 = hf(t_i, u_i), \quad v = u_i + \frac{1}{2}k_1.$$

The second stage employs an Euler step over the who time step using the value of the slope from the first stage:

$$k_2 = hf(t_i + \frac{1}{2}h, v), \quad u_{i+1} = u_i + k_2.$$

3. (a) (15 points) Write a matlab function, `hw10p3ie(f, a, b, u0, n)`, that accept the function in `u'(t)=f(t,u)`, the endpoints of time interval `a` and `b`, the initial value `u0`, and the number of time steps `n`, and returns the vector of nodes `t` and the vector of solution values `u`.

   (b) (15 points) Write a matlab script, **hw10p3**, that for $n = 20 \cdot 2^k$, where $k = 1, 2, \ldots 9$ calculates the absolute value of the error for the solution of the equation `u' = u` between $a = 0$ and $b = 1$, `u0 = 1`. Plot the graph of the error of your solution, δ, vs. n in double logarithmic axes. In addition, as guides, plot the graphs of $\delta(n) = 1/n^l$, for $l = 1, 2, 3, 4$. Analyze your figure and by visual inspection determine the dependence $\delta \sim n^\gamma$. Describe your results in gitlab's README.md file.

Gitlab

4. (5 points) Create a gitlab project called **hw10** (name it exactly as shown). Upload **all** matlab files that are required to run your code. Share the project with the instructor and the TAs and grant them **Reporter** privileges.