

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Section: \_\_\_\_\_

Collaborators: \_\_\_\_\_

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	35	30	25	5	95
Score:					

**Instructor/grader comments:**

1. Two ladders crisscross an alley of unknown width  $W$ . Each ladder reaches from the base of one wall to some point on the opposite wall. The ladders cross at a height  $H$  above the pavement. Find  $W$  given that the lengths of the ladders are  $L_1 = 20$  and  $L_2 = 30$  and that  $H = 8$ . (Lengths are in some arbitrary units.)

- (a) (10 points) Derive the equation which root gives the value of  $W$ ,

$$f(W, L_1, L_2, H) = 0.$$

Write your derivation in the space below.

Hints: the triangles ABE and DBC are similar; so are the triangles ABF and ADC.

- (b) (5 points) Write a matlab function, `hw07p1alley(w, l1, l2, h)`, that returns the values of  $f(W, L_1, L_2, H)$ . Place it in its own matlab file. Provide a help string for your function.
- (c) (20 points) Write a matlab script (call it e.g. **hw07p1.m**) that (a) initializes the values of the parameters  $L_1$ ,  $L_2$ , and  $H$ ; (b) prints the help text for `hw07p1alley` (c) defines an anonymous function of a single variable,  $f(w)$ , that itself uses `hw07p1alley(w, l1, l2, h)`; (c) solves the equation  $f(w) = 0$  using (i) the bisection and (ii) the secant methods.

Use the code that we developed in class. Place the commands `clear`, and `format compact` at the top of your script.

Compare the performance of the methods in the project's README file.

Hint: since  $0 \leq W \leq \min(L_1, L_2)$  it is reasonable to choose exactly those boundaries for the bisection method.

2. (a) (10 points) Write a matlab function, `hw07p2newtm(F, Fprime, Fdprime, x0, itermax)`, that implements the algorithm you developed in HW6 Problem 2 and returns the vector of the root approximation and the number of function evaluations that was used to find the root. (Count together the number of evaluations of the function and its derivatives.) Provide a help string for your function.
- (b) (15 points) Write a matlab script (call it e.g. **hw07p2.m**) that tests your algorithm using the following function:

$$f(x) = \sin^4(x).$$

Find the expressions for  $f'(x)$  and  $f''(x)$ . Use the initial approximation for the root  $x_1 = 3$ .

Compare your results with the results (both for the value of the root and the number of function evaluations) of “unmodified” Newton's algorithm. Use the code for the Newton's algorithm that we developed in class.

- (c) (5 points) Describe your results in the project's README.md file.
3. An algorithm for finding the roots of nonlinear equation uses three function evaluations per iteration step and claims the following relations between the errors of successive steps:

$$\epsilon_{n+1} = \alpha \epsilon_n \sqrt{\epsilon_{n-1}},$$

where  $\alpha$  is a positive constant.

- (a) (20 points) What is the convergence rate of the algorithm (per iteration)? Show your calculations in the space below. Arrange the algorithms from the fastest to the slowest (per iteration): the “mystery” algorithm, bisection, secant, newton’s.

- (b) (5 points) What is the convergence rate of the algorithm (per function evaluation)? Show your calculations in the space below. Arrange the algorithms from the fastest to the slowest (per function evaluation): the “mystery” algorithm, bisection, secant, newton’s.

**Gitlab**

4. (5 points) Create a gitlab project called **hw07** (name it exactly as shown). Upload **all** matlab files that are required to run your code. Share the project with the instructor and the TAs and grant them **Reporter** privileges.