A PLANE TRUSS

http://www.phys.uconn.edu/~rozman/Courses/m3510_17f/

Last modified: September 19, 2017

Figure 1 depicts a plane truss having nine members (the numbered lines) connecting six joints (the numbered circles). The indicated loads are applied at joints I and III, and we want to determine the resulting forces, f_i, $i = 1, \ldots, 9$, on each member of the truss.

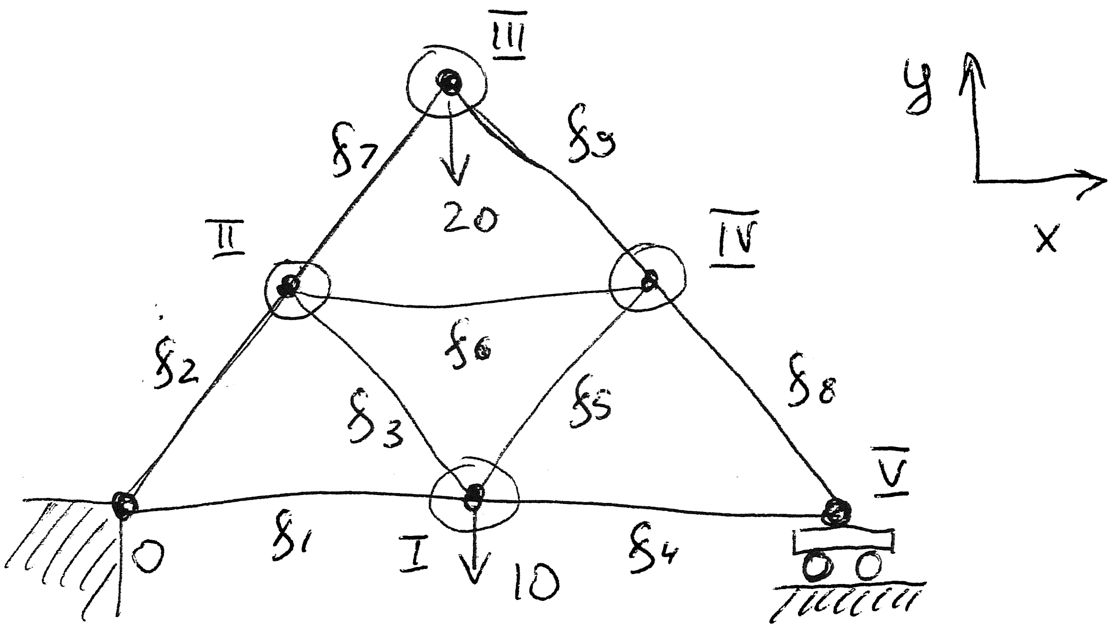

Figure 1: A plane truss

For the truss to be in static equilibrium, there must be no net force, horizontally or vertically, at any joint. Thus, we can determine the member forces by equating the horizontal forces to the left and right at each joint, and similarly equating the vertical forces upward and downward at each joint.

For the six joints, this would give 12 equations, which is more than the nine unknown factors to be determined. For there to be a unique solution, we assume that joint 0 is rigidly fixed both horizontally and vertically and that joint V is fixed vertically. Resolving the member forces into horizontal and vertical components and defining $\alpha = \frac{1}{\sqrt{2}}$, we obtain the following system of equations for the member forces f_i:

Joint I:

$$\begin{aligned} f_1 + \alpha f_3 &= f_4 + \alpha f_5 \\ \alpha f_3 + \alpha f_5 &= 10 \end{aligned} \tag{1}$$

Joint II:

$$\begin{aligned} \alpha f_2 &= \alpha f_3 + f_6 + \alpha f_7 \\ \alpha f_2 + \alpha f_3 &= \alpha f_7 \end{aligned} \tag{2}$$

Joint III:

$$\begin{aligned} \alpha f_7 &= \alpha f_9 \\ \alpha f_7 + \alpha f_9 &= 20 \end{aligned} \tag{3}$$

Joint IV:

$$\begin{aligned} \alpha f_6 + \alpha f_5 + \alpha f_9 &= \alpha f_8 \\ \alpha f_5 + \alpha f_8 &= \alpha f_9 \end{aligned} \tag{4}$$

Joint V:

$$f_4 + \alpha f_8 = 0$$

In matrix notations,

$$\begin{bmatrix} 1. & 0. & \alpha & -1. & -\alpha & 0. & 0. & 0. & 0. \\ 0. & 0. & \alpha & 0. & \alpha & 0. & 0. & 0. & 0. \\ 0. & \alpha & -\alpha & 0. & 0. & -1. & -\alpha & 0. & 0. \\ 0. & \alpha & \alpha & 0. & 0. & 0. & -\alpha & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & \alpha & 0. & \alpha \\ 0. & 0. & 0. & 0. & 0. & 0. & \alpha & 0. & \alpha \\ 0. & 0. & 0. & 0. & \alpha & \alpha & 0. & -\alpha & \alpha \\ 0. & 0. & 0. & 0. & \alpha & 0. & 0. & \alpha & -\alpha \\ 0. & 0. & 0. & 1. & 0. & 0. & 0. & \alpha & 0. \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} = \begin{bmatrix} 10. \\ 0. \\ 0. \\ 0. \\ 0. \\ 20. \\ 0. \\ 0. \\ 0. \end{bmatrix}.$$