Name: _____

Date: _____

Collaborators:

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	5	6	Total
Points:	20	10	10	20	10	30	100
Score:							

Instructor/grader comments:

First order linear differential equations

- 1. Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \rightarrow \infty$.
 - (a) (10 points)

$$\frac{\mathrm{d}y}{\mathrm{d}t} - 2y = t^2 e^{2t}$$

(b) (10 points)

$$t\frac{\mathrm{d}y}{\mathrm{d}t} - y = t^2 e^{-t}, \quad t > 0$$

2. (10 points) Find the solution of the following initial value problem.

$$\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0$$

3. (10 points) The behavior of solutions of the following equation, as $t \to +\infty$, qualitatively depends on the choice of the initial value y(0) = a. Determine the value of *a* for which the transition from one type of behavior to another occurs.

$$3\frac{\mathrm{d}y}{\mathrm{d}t} - 2y = e^{-\pi t}, \quad y(0) = a$$

Separable equations

- 4. Find the general solution of the given differential equation.
 - (a) (10 points)

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 2\frac{x^2}{y} = 0$$

(b) (10 points)

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x - e^{-x}}{y + e^y} = 0$$

5. (10 points) Solve the given initial-value problem, and determine the interval of existence of the solution.

$$\cos y \frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{t\sin y}{1+t^2}, \quad y(1) = \frac{\pi}{2}$$

Homogeneous equations

If the right-hand side of the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

can be expressed as a function of the ratio y/x only, then the equation is said to be homogeneous. Such equations can always be transformed into separable equations by a change of the dependent variable.

6. Consider the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y - 4x}{x - y}$$

- (a) (5 points) Show that the equation is homogeneous, i.e. that right-hand side of the equation can be expressed as a function of the ratio y/x only.
- (b) (10 points) Introduce a new dependent variable v so that v = y/x, or y = xv(x). Express dy/dx in terms of x, v, and dv/dx.
- (c) (5 points) Replace y and dy/dx in the equation by the expressions from part (b) that involve v and dv/dx. Find the differential equation for v(x) and observe that equation is separable.
- (d) (10 points) Solve the equation obtaining v implicitly in terms of x. Find the solution of the original equation by replacing v with y/x.