# SPHERICALLY-SYMMETRIC DEFORMATIONS

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https://www.phys.uconn.edu/~rozman/Courses/P3102_26S/

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In spherical polar coordinates $r$, $\theta$, $\varphi$:

$$\mathbf{u} = (u_r(r),\ 0,\ 0), \tag{1}$$

$$\mathbf{f} = (f_r(r),\ 0,\ 0). \tag{2}$$

Navier-Cauchy equation of equilibrium:

$$(2\mu + \lambda)\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 u_r\right)\right) = -f_r(r), \tag{3}$$

Strain tensor:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} = \frac{u_r}{r}, \tag{4}$$

$$\varepsilon_{\theta\varphi} = \varepsilon_{r\theta} = \varepsilon_{\varphi r} = 0. \tag{5}$$

Hooke's law:

$$\sigma_{rr} = 2\mu\varepsilon_{rr} + \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right), \tag{6}$$

$$\sigma_{\theta\theta} = 2\mu\varepsilon_{\theta\theta} + \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right), \tag{7}$$

$$\sigma_{\varphi\varphi} = 2\mu\varepsilon_{\varphi\varphi} + \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right). \tag{8}$$