HW10

due April 23, 2026

Show all your work and indicate your reasoning in order to receive the credit. That is, answers without derivations do not earn credit.

To solve the problems, use the methods that are taught in this class. If a problem statement includes directions for solutions, following those directions is required. If you found a different method of solution, you are welcome to include it for extra credits.

Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	Total
Points:	70	70
Score:		

Instructor/grader comments:

1. (70 points) A drop of viscous, incompressible fluid with volume V and viscosity η is squeezed between two parallel plates (see Fig. 1). While the bottom plate remains stationary, the top plate moves toward it at a constant velocity u. Determine the force $F(u, h, \eta, V)$ exerted on the top plate.

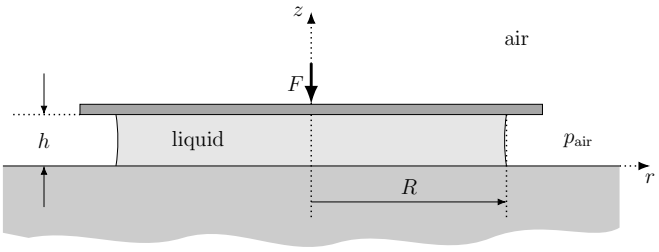

Figure 1: A drop of fluid squeezed between two parallel plates.

Assumptions:

- The flow is cylindrically symmetric; the fluid forms a disk of thickness $h(t)$ and radius $R(t)$ at all times, where $R \gg h$

$$R(t) = \sqrt{\frac{V}{\pi h(t)}}. \tag{1}$$

- The advective terms in the Navier-Stokes equation may be neglected.

Directions:

(a) Specify the boundary conditions for the radial component, v_r, and the axial component, v_z, of the flow velocity.

(b) Use the cylindrical coordinate system as shown in Fig. 1. For reference, the continuity equation in cylindrical coordinates is:

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0. \tag{2}$$

(c) Explain the validity of any further simplifications or assumptions you make regarding the Navier-Stokes equation (beyond the omission of advective terms).

(d) Derive the expression for the radial velocity component of the flow, v_r.

(e) Use the continuity equation to determine the pressure distribution within the fluid.

(f) Calculate the total force $F(u, h, \eta, V)$ exerted on the top plate using the pressure distribution. Provide a clear explanation of your reasoning.