

**HW9**

due April 16, 2026

Show all your work and indicate your reasoning in order to receive the credit. That is, answers without derivations do not earn credit.

To solve the problems, use the methods that are taught in this class. If a problem statement includes directions for solutions, following those directions is required. If you found a different method of solution, you are welcome to include it for extra credits.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Collaborators: \_\_\_\_\_

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	Total
Points:	30	40	70
Score:			

**Instructor/grader comments:**

**Simple viscous flows**

1. (30 points) Industrial coating processes can be modeled by a simple setting where an infinite plate is vertically withdrawn with a constant speed  $u$  from a pool of viscous liquid of viscosity  $\nu$ . Given that, eventually, a steady state planar flow is established, find the thickness of the liquid film that the plate carries away from the pool,  $h(u, \nu, g)$  where  $g$  is the acceleration of gravity.
2. (40 points) Calculate the velocity profile of a pressure-driven laminar viscous flow in a long cylindrical pipe of elliptical cross-section:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (1)$$

The length of the pipe is  $L$ , the pressure drop is  $\Delta p$ , the viscosity of the fluid is  $\nu$ .

**Directions:**

- (a) Use Cartesian coordinate system with the  $z$ -axis along the axis of the cylinder.
- (b) Search for the solution of Poisson's equation in the form of quadratic polynomial in  $x$  and  $y$ . Keep the terms with the even powers of coordinates. (Why?)