HW8

due April 9, 2026

Show all your work and indicate your reasoning in order to receive the credit. That is, answers without derivations do not earn credit.

To solve the problems, use the methods that are taught in this class. If a problem statement includes directions for solutions, following those directions is required. If you found a different method of solution, you are welcome to include it for extra credits.

Name: ________________________

Date: ________________________

Collaborators: ____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	Total
Points:	30	40	70
Score:			

Instructor/grader comments:

1. (30 points) Find the ratio of the frequencies of small oscillations for a pendulum in two cases: first, in a vacuum (or air, neglecting resistance), and second, when the pendulum bob is completely immersed in an ideal fluid of density ρ_f. Assume the bob is a sphere with density ρ_b, where $\rho_b > \rho_f$.

2. (40 points) A long half-cylinder of radius R lies flat on the bottom of a pool filled with an ideal fluid of density ρ. The cylinder's axis is aligned with the z-axis. A uniform fluid flow with velocity U is directed perpendicular to the axis of the cylinder, along the positive x-direction (see Figure 1).

   Neglecting the effects of gravity and assuming the pressure at infinity is zero, calculate the magnitude and direction of the force exerted by the fluid on the half-cylinder per unit length.

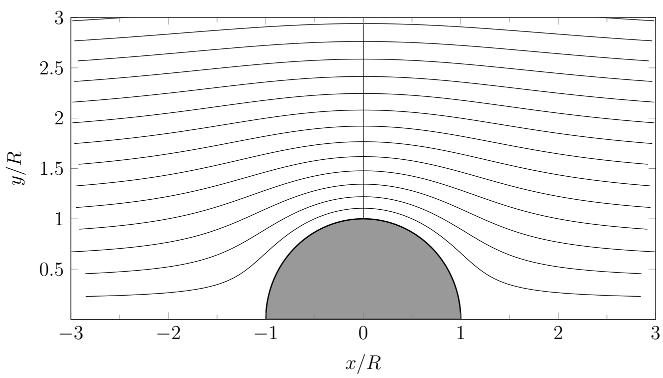

Figure 1: A sketch of streamlines of an asymptotically uniform cross-flow around a half-cylinder.

   Directions:

   (a) Recall that the velocity potential $\Psi(r,\phi)$ for a uniform cross-flow with velocity U around a full circular cylinder of radius R (in polar coordinates with the origin at the center) is:

   $$\Psi(r,\phi) = U r \cos\phi \left(1 + \frac{R^2}{r^2}\right), \tag{1}$$

   where ϕ is the polar angle measured from the direction of the flow. The velocity field is given by $\mathbf{v} = \nabla\Psi$.

   (b) Establish the relationship between the velocity potential for flow around a full circular cylinder, as given in Eq. (1), and the velocity potential required for the flow around the half-cylinder in this problem.

   (c) Determine the pressure distribution at the surface of the half-cylinder.

   (d) Calculate the net force per unit length. Verify the dimension of your final answer.

   (e) Estimate the magnitude of the force acting on a 1-meter segment of the cylinder using the following parameters: $\rho = 10^3$ kg/m^3, $U = 2$ m/s, $R = 0.1$ m.