HW7
due March 26, 2026

Show all your work and indicate your reasoning in order to receive the full credit.

Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	Total
Points:	20	40	60
Score:			

Instructor/grader comments:

1. (20 points) In a water clock (see Figure 1), the water level h in the vessel serves as the time indicator. To ensure the clock remains accurate, the height must decrease at a constant rate:

$$\frac{dh}{dt} = C. \tag{1}$$

Determine the shape of the vessel by finding the functions $r(h)$ and $h(r)$, where r is the radius of the vessel at any given height h.

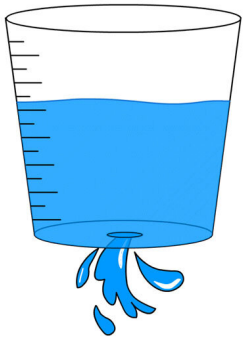

Figure 1: Sketch of the flow in water clocks

2. (40 points) A vertical cylinder with cross-sectional area A and negligible mass is placed on a long, straight, horizontal, frictionless rail. The cylinder has a narrow spout of cross-sectional area a ($a \ll A$) at its base. The spout is oriented horizontally and parallel to the rail. The container is initially at rest and is filled with fluid to a height H. When the plug in the spout is removed, the fluid streams out, accelerating the container in the opposite direction.

   (a) Find the time T required for the fluid to drain completely. Note that the container accelerates only while the fluid is draining.

   (b) Using the conservation of momentum, one can derive the following relation:

$$m\,\mathrm{d}v = -u\,\mathrm{d}m, \tag{2}$$

   where m is the mass of the fluid remaining in the container, u is the velocity of the fluid leaving the spout relative to the container, v is the velocity of the container relative to the "rail" reference frame, $\mathrm{d}m$ is the change in the mass of the fluid within the container over a time interval $\mathrm{d}t$ ($\mathrm{d}m < 0$).

   Use this equation to find the acceleration of the container. Notice that the acceleration is constant:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \text{const}. \tag{3}$$

   (c) Use your derived values for T and $\frac{\mathrm{d}v}{\mathrm{d}t}$ to find the terminal velocity of the container.