HW6

due March 12, 2026

Show all your work and indicate your reasoning in order to receive the full credit.

Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	3	Total
Points:	30	20	30	80
Score:				

Instructor/grader comments:

Slender beams

1. (30 points) A rectangular sheet of paper has length a, width b, and thickness d, where $a > b \gg d$. When placed on two simple supports, the paper sags noticeably under its own weight. Now, consider the same paper rolled into a cylindrical tube of length a and radius $r = b/2\pi$.

   Assuming the arrangement of the supports remains identical in both cases, find the ratio of the maximum vertical deflections for these two configurations.

   Estimate this ratio for a letter-size paper sheet. Look up the necessary dimensions, or provide reasonable estimates.

Inertial cavitation

2. (20 points) The process by which a bubble in a liquid rapidly collapses, producing a shock wave, is known as *inertial cavitation*. This phenomenon occurs in both natural environments and engineering applications. Understanding the dynamics of the collapse — specifically the time required for the bubble to close — is a fundamental step in the study of cavitation.

   (a) Use dimensional analysis to find the collapse time of a spherical cavity with an initial radius a, formed within an ideal, incompressible fluid of density ρ and ambient pressure p.

   (b) By what factor does the collapse time change if the initial radius of the cavity is doubled and the fluid pressure is quadrupled?

Conservation of mass

3. (30 points) A vertical cylindrical pipe of internal radius R is filled with an incompressible fluid of density ρ. A long cylinder of negligible mass, length L, ($L \gg R$), and outer radius $r = R - \delta$, ($0 < \delta \ll R$) is immersed in the fluid.

   If the cylinder moves vertically with velocity u, find the total kinetic energy of the system, E_k, and determine the *effective mass* of the cylinder.

   Assume that:

   (a) The cylinder and pipe remain co-axial at all times.

   (b) Edge effects are negligible due to the cylinder's length.

   (c) The fluid counterflow is uniform through the annular region between the cylinder and the pipe.