HW3

due February 12, 2026

Show all your work and indicate your reasoning in order to receive the full credit.

Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	3	Total
Points:	10	15	15	40
Score:				

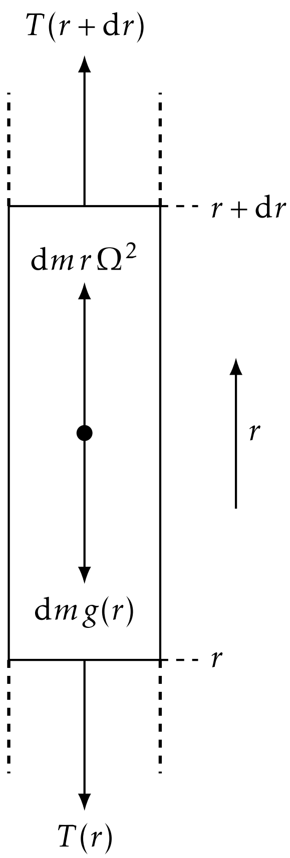
Instructor/grader comments:

Breaking length of a material

1. (10 points) A *space elevator* (see https://en.wikipedia.org/wiki/Space_elevator for some background information) can be created if it becomes technically feasible to lower a line down to Earth from a geostationary satellite. The goal of the problem is to find the requirements for the *breaking length* of the line's material.

   Breaking length, L, is a measure of a material's *tensile strength*, $\sigma_{\rm cr}$, defined as is the maximum length of a vertical column of the material (assuming a fixed cross-section) that could suspend its own weight when supported only at the top. For this measurement, the definition of weight is the force of gravity at the Earth's surface applying to the entire length of the material, not diminishing with height: $L \equiv \frac{\sigma_{\rm cr}}{g_0 \rho}$ where g_0 is the acceleration of gravity at the surface of the Earth.

   For the purpose of this problem, assume that (i) the line is unstretchable and has constant cross-section A and constant density ρ, (ii) the satellite is located above the equator, (iii) the Earth is an ideal sphere.



   Parts (a), (b), and (c) of this problem to be discussed in class.

   Work in the (non-inertial) Earth-based reference frame. In this reference frame the external forces acting on the line are the gravitational attraction directed toward the center of the Earth and the centrifugal force directed away from the center of the Earth. (See the figure on the left.) In particular, a geostationary satellite is located at the point where the two forces balance each other.

   (a) Find the radius of a geostationary orbit, r_s. Verify the dimensions of your answer. Find numerical value of r_s. Notice that $r_s \gg r_e$.
   The radius of the Earth $r_e \approx 6.4 \times 10^6$ m, the angular frequency of the Earth' rotation $\omega = \frac{2\pi}{24\,\text{hours}} \approx 7.27 \times 10^{-5}\ \text{s}^{-1}$.

   (b) Derive the first order differential equation for the force of tension in the line, $T(r)$. Find its solution for the initial condition $T(r_e) = 0$.

   (c) Calculate tension in the line, $\sigma(r) = T(r)/A$. Find the maximal tension. When doing the calculations, neglect terms $\sim r_e$ when compared with terms $\sim r_s$ and neglect terms $\sim 1/r_s$ when compared to $\sim 1/r_e$.

   (d) Find the value of the required breaking length. What materials can be used for construction of the line?

Strain tensor

2. (15 points) Calculate the strain tensor for the displacement field

$$\vec{\mathbf{u}} = (Ax_1 + Cx_2, Cx_1 - Bx_2, 0),$$

where A, B, C are (small) constants. Under what conditions the volume is unchanged?

3. (15 points) A displacement field is given by

$$\begin{aligned} u_1 &= \alpha(x_1 + 2x_2) + \beta x_1^2 \\ u_2 &= \alpha(x_2 + 2x_3) + \beta x_2^2 \\ u_3 &= \alpha(x_3 + 2x_1) + \beta x_3^2. \end{aligned}$$

where α and β are small. Calculate the strain tensor.