- (a) Show all your work and indicate your reasoning in order to receive the credit.
- (b) Present your answers in *low-entropy* form.
- (c) Use words and pictures to supplement your equations
- (d) Work must progress linearly down the page recopy solutions that are too nonlinear
- (e) Box your final answer(s) and important intermediate results.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Question:	1	2	3	Total
Points:	35	35	30	100
Score:				

Instructor/grader comments:

1. (35 points) Two parallel circular disks (of radius  $R_0$ ) lie one above the other small distance d,  $d \ll R_0$  apart (see Fig. 1). The space between the disks is filled with incompressible viscous fluid of the dynamic viscosity  $\eta$ . The 'bottom' disk is at rest while the 'top' one moves toward the 'bottom' disk (thus displacing the fluid) with the velocity u. Find the force applied to the 'top' disk,  $F = F(u, d, \eta, R_0)$ .



Figure 1: Sketch of the flow

2. (35 points) A sphere of radius *R* and negligible mass (its density is small in comparison with the density of the surrounding fluid) is immersed in a container filled with an ideal incompressible fluid of density  $\rho$ . The sphere is held inside the fluid (very far from the bottom, the side walls or the free surface of the fluid) by a long elastic spring (spring constant *k*) attached to the bottom of the container. Find the frequency of the small vertical oscillations of the sphere about its equilibrium position.

Hint: the velocity potential for a flow around a sphere of radius *R* is:

$$\psi(r,\theta) = Ur\left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos(\theta),$$

where *U* is the velocity of the undisturbed flow at infinity.

3. (30 points) A drop of inviscid incompressible fluid of the volume V and the density  $\rho$  is squeezed between two parallel smooth plates. The 'bottom' plate is at rest while the 'top' one moves toward the 'bottom' plate with the velocity *u*. Find the force applied to the 'top' plate,  $F = F(u, h, \rho, V)$ , assuming that the fluid thickness *h* is very small compared with the liquid's radial extent.