(a) Show all your work and indicate your reasoning in order to receive the credit.

(b) Present your answers in *low-entropy* form.

(c) Use words and pictures to supplement your equations

(d) Work must progress linearly down the page – recopy solutions that are too nonlinear

(e) $\boxed{\text{Box}}$ your final answer(s) and important intermediate results.

Name: ______________________________

Date: ______________________________

Question:	1	2	3	4	5	Total
Points:	30	10	20	20	20	100
Score:						

Instructor/grader comments:

Thin-wall containers

1. (30 points) Can a helium balloon lift the tank used to transport its helium gas?

   Hints: assume that

   (a) the helium pressure inside the tank, P, is much higher than the atmospheric pressure, P_0: $P \gg P_0$, but still is low enough to treat the helium in the tank as an ideal gas

   (b) the density of the air, ρ_a, is much larger than the density of the helium, ρ_h: $\rho_a \gg \rho_h$

   (c) the helium tank is a spherical container with thin walls made of steel

   (d) the weight of the material of the balloon is much smaller than the weight of the tank

   For the numerical estimates use $\rho_a \approx 1.2$ kg/m^3, $P_0 \approx 10^6$ Pa, the density of the steel $\rho_s \approx 8000$ kg/m^3, and the yield stress for the steel $\sigma_{\text{yield}} \approx 2.5 \times 10^8$ Pa.

Self-gravitating non-rotating spheres

2. (10 points) Compare the pressure in the center of self-gravitating sphere when the material of the sphere is an incompressible fluid and when it is an elastic material with Young's modulus, E, and the Poisson's ratio, ν.

   Hints:

   (a) use the expressions for the pressure we found in class

   (b) in order to have both problem equivalent, you need to properly chose ν.

Material derivatives

3. Consider an incompressible steady flow in a stream with constant depth, $z = d$, bounded on one side by a straight wall $y = 0$, and on the other side by a curve $y = h(x)$, which is a slowly varying function, $\frac{\mathrm{d}h}{\mathrm{d}x} \ll 1$.

   (a) (10 points) calculate the acceleration in the flow

   (b) (10 points) What should $h(x)$ be in order for the acceleration to be independent of x?

Newton's bucket

4. (20 points) The surface of an incompressible liquid in a steady-rotating cylinder of radius R with its axis in the vertical direction takes the form of a paraboloid of revolution. What should be the angular frequency of rotation such that the surface of the fluid 'touches' the bottom of the cylinder? How high is the fluid at the rim of the cylinder? The volume of the liquid in the cylinder, $V = \pi r^2 H$.

Dimensional analysis

5. The process in which a bubble in a liquid rapidly collapses, producing a shock wave, is called *inertial cavitation*. Inertial cavitation occurs in nature as well as in man-made objects and processes. Understanding the dynamics of the collapse, in particular the time to collapse, is an important step in understanding the cavitation.

   (a) (15 points) Use the dimensional analysis to find the the collapse time of an spherical cavity of initial radius a formed in the bulk of an ideal incompressible fluid of density ρ and pressure p.

   (b) (5 points) How the collapse time is going to change if you double the initial radius of the sphere and quadruple the pressure?