Name: ______________________

Date: ______________________

Collaborators: ______________________

(Collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	50	20	20	90
Score:				

Instructor/grader comments:

Geometry of planar curves

1. (a) (15 points) Show that the radius of curvature, R, of a curve $y = y(x)$ is

$$\frac{1}{R} = \frac{y''(x)}{[1 + y'(x)^2]^{\frac{3}{2}}}. \tag{1}$$

Hint: Start your derivations from the equations describing a planar curve using the arc length, s, and the elevation angle, θ:

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{1}{R}, \quad \frac{\mathrm{d}x}{\mathrm{d}s} = \cos\theta, \quad \frac{\mathrm{d}y}{\mathrm{d}s} = \sin\theta.$$

Find the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of θ and calculate its derivative with respect to x (thus finding $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$).

(b) (10 points) Use Eq. (1) to calculate the radius of curvature of the curve $y = \cosh(x)$.

(c) (15 points) Use Eq. (1) to show that the radius of curvature, R, of a curve parametric curve $x = x(t)$, $y = y(t)$ is

$$\frac{1}{R} = \frac{y''(t)\,x'(t) - x''(t)\,y'(t)}{[x'(t)^2 + y'(t)^2]^{\frac{3}{2}}}. \tag{2}$$

(d) (10 points) Use Eq. (2) to calculate the radius of curvature of a cycloid specified by the following parametric equations:

$$\begin{aligned} x(t) &= \rho\,(t - \sin(t)) \\ y(t) &= \rho\,(1 - \cos(t)) \end{aligned}$$

for $0 \le t \le \pi$.

Slender beams

2. (20 points) A beam of rectangular crossection $a \times b$ is cut out of a circular cylinder of radius R. The beam placed on a frictionless horizontal table. One end of the beam is fixed; a force F is applied horizontally and perpendicularly to the beam at the other end. Find a and b of the beam which has the smallest deflection at the free end.

3. (20 points) A piece of paper of length a, width b and thickness d, when placed on two simple supports, noticeably sags down under its own weight. But when the paper rolled into a tube, it becomes much harder to bend.

Let the maximal deflection of the plane piece of paper under its own weight be δ_1, while the equivalent deflection of the paper tube be δ_2. Find $\frac{\delta_1}{\delta_2}$.