Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	Total
Points:	15	35	50
Score:			

Instructor/grader comments:

1. (15 points) A stress tensor is given by (the units are Pa):

$$\sigma = \begin{pmatrix} 100 & 100 & 160 \\ 100 & 0 & -150 \\ 160 & -150 & -60 \end{pmatrix}.$$

For the surface element with the unit normal vector

$$\vec{\mathbf{q}} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

find the stress vector $\vec{\mathbf{p}}_{\mathbf{q}}$ and the magnitude of its tangential and normal components. Find the angle between $\vec{\mathbf{p}}_{\mathbf{q}}$ and $\vec{\mathbf{q}}$.

2. A space elevator (see the Wikipedia article Space_elevator for some background information) can be created if it becomes technically feasible to lower a line down to Earth from a geostationary satellite. Assume that (i) the line is unstretchable and has constant cross-section A and constant density ρ, (ii) the satellite is located above the equator, (iii) the Earth is an ideal sphere. Assume that you know the radius of the Earth R, its angular frequency of rotation Ω, and the acceleration of the gravity at the surface of the Earth g.

   (a) (10 points) Find the height of a geostationary satellite. Verify the dimensions of your answer

   (b) (15 points) Calculate tension in the line, $\sigma(r)$. Find the maximal tension.

   (c) (10 points) Estimate the numerical value of the ratio of maximal tension to line's density. Compare with the tensile strength (breaking tension) for steel: $\sigma_t \approx 4 \times 10^6$ Pa, $\rho \approx 8 \times 10^3$ kg/m^3.

   Hint: work in the (non-inertial) Earth-based reference frame. In this reference frame the external forces acting on the line are the gravitational attraction (directed toward the center of the Earth) and the centrifugal force (directed away from the center of the Earth). In particular, a geostationary satellite is located at the point where the two forces balance each other.