

In order to receive the most credit show all your work and indicate your reasoning clearly.

1. (Dynamics of ideal fluids:) A large container is filled with an ideal incompressible fluid of the density ρ . A small sphere of radius a and a negligible mass is held inside the fluid by a long elastic spring (spring constant k) attached to the bottom of the container. Find the frequency of the small vertical oscillations of the sphere about its equilibrium position.
2. (Dynamics of viscous fluids:) A vertical pipe of radius R is filled up with a viscous incompressible fluid of the density ρ and viscosity η . A long cylinder of length L , $L \gg R$ and a negligible mass is immersed co-axially into the pipe so that only a small gap of width h , $h \ll R$ is formed between their lateral surfaces. The system is placed in a uniform gravitational field of acceleration g and the cylinder is released. Find the steady state velocity of the cylinder.
3. (Large deformation of beams:) A longbow is constructed from a rectangular steel beam of length $L = 2$ m with dimensions $2a = 10$ mm and $2b = 20$ mm. The longbow is bent such that its ends are parallel. It is kept in equilibrium by the tension in the bowstring. Find the shape of the bow, the length of the bowstring, and the tension in the string. (Young's modulus of steel is $E = 2 \times 10^{11}$ Pa.)