12.30. **Maximum mass**

The energy of the resulting particle is E. Let its mass be M and its momentum be p_f. Then the very important relation gives $E^2 = p_f^2 + M^2$. Since E is given, M is maximum when $p_f = 0$. That is, the initial momenta are equal and opposite. Call them p. Then the sum of the energies of the photon and initial mass is

$$E = p + \sqrt{p^2 + m^2} \implies (E-p)^2 = p^2 + m^2 \implies p = \frac{E^2 - m^2}{2E}. \qquad (696)$$

The energy of the photon is therefore

$$E_\gamma = p = \frac{E^2 - m^2}{2E} \longrightarrow \frac{E^2 - m^2c^4}{2E}. \qquad (697)$$

The energy of the mass is then

$$E_m = E - E_\gamma = \frac{E^2 + m^2c^4}{2E}. \qquad (698)$$

If $m \approx 0$, then $E_\gamma \approx E_m \approx E/2$ (we essentially have two photons). If $m \approx E/c^2$, then $E_\gamma \approx 0$ and $E_m \approx E$ (both momenta are small).

12.31. **Equal angles**

Conservation of p_y says that the y components of the two final momenta are equal and opposite. The equality of the two angles then implies that the p_x components are equal. Conservation of p_x then says that both p_x's are equal to $E/2$. Both momenta therefore have magnitude $E/(2\cos\theta)$.

Conservation of energy gives the final energy of m as $E_m = E + m - E/(2\cos\theta)$. The very important relation applied to m then gives

$$\left(E + m - \frac{E}{2\cos\theta}\right)^2 = \left(\frac{E}{2\cos\theta}\right)^2 + m^2 \implies \cos\theta = \frac{E+m}{E+2m} \longrightarrow \frac{E+mc^2}{E+2mc^2}. \qquad (699)$$

In the limit $E \ll mc^2$, we have $\cos\theta \approx 1/2 \implies \theta \approx 60^\circ$ (not obvious). In the limit $E \gg mc^2$, we have $\cos\theta \approx 1 \implies \theta \approx 0^\circ$.

12.32. **Pion-muon race**

We are given $\gamma mc^2 = 10$ GeV for both particles. Using $m_\pi c^2 \approx 137$ MeV and $m_\mu c^2 \approx 105.7$ MeV, we find $\gamma_\pi \approx 73.0$ and $\gamma_\mu \approx 94.6$. Now,

$$\gamma \equiv 1/\sqrt{1 - v^2/c^2} \implies v = c\sqrt{1 - 1/\gamma^2} \approx c(1 - 1/2\gamma^2), \qquad (700)$$

for reasonably large γ. The difference in the two speeds is therefore $\Delta v \approx c(1/2\gamma_\pi^2 - 1/2\gamma_\mu^2)$. The total time is essentially $t \approx (100\,\text{m})/c$, so the distance the pion lags behind the muon after this time is

$$\Delta d = t\Delta v \approx \frac{100\,\text{m}}{c} \cdot c\left(\frac{1}{2(73.0)^2} - \frac{1}{2(94.6)^2}\right) \approx 3.8 \cdot 10^{-3}\,\text{m} = 3.8\,\text{mm}. \qquad (701)$$