11.52. **Running away**

In A's frame, the mark on the ground starts a distance $(4/5)L$ away from A and moves toward her at speed $3c/5$. So the time this takes is $(4L/5)/(3c/5) = 4L/3c$. In A's frame, B's speed is

$$V = \frac{\frac{3c}{5} + \frac{3c}{5}}{1 + \left(\frac{3}{5}\right)^2} = \frac{15c}{17}\,. \tag{632}$$

So B travels a distance $(15c/17)(4L/3c) = (20/17)L$ by the time the mark reaches A. If we work with a general v instead of $3c/5$, the answer to the problem is $2L/(\gamma(1+v^2))$. Note that this is less than $2L/\gamma$, so B has not yet reached a mark at $-L$ at this time.

11.53. **Angled photon**

Using $v'_x = c\cos\theta$ and $v'_y = c\sin\theta$, the velocity-addition formulas give the velocity components in S as

$$v_x = \frac{v + c\cos\theta}{1 + (v/c)\cos\theta}\,, \qquad \text{and} \qquad v_y = \frac{c\sin\theta}{\gamma_v\big(1 + (v/c)\cos\theta\big)}\,. \tag{633}$$

So we have

$$\begin{aligned}
v_x^2 + v_y^2 &= \left(\frac{v + c\cos\theta}{1 + (v/c)\cos\theta}\right)^2 + \left(\frac{v + c\sin\theta}{\gamma_v\big(1 + (v/c)\cos\theta\big)}\right)^2 \\
&= \frac{c^2}{(c + v\cos\theta)^2}\left((v + c\cos\theta)^2 + \left(1 - \frac{v^2}{c^2}\right)(c\sin\theta)^2\right) \\
&= \frac{c^2}{(c + v\cos\theta)^2}\Big(v^2(1 - \sin^2\theta) + 2vc\cos\theta + c^2(\cos^2\theta + \sin^2\theta)\Big) \\
&= \frac{c^2}{(c + v\cos\theta)^2}\big(v^2\cos^2\theta + 2vc\cos\theta + c^2\big) \\
&= c^2.
\end{aligned} \tag{634}$$

12.21. **System of particles**

Let the CM move with velocity v with respect to the lab frame. Then the Lorentz transformation for the total momentum is $p^{\rm CM}_{\rm total} = \gamma_v(p^{\rm lab}_{\rm total} - (v/c^2)E^{\rm lab}_{\rm total})$. The minus sign here is due to the fact that the CM frame sees the lab frame move with velocity $-v$. Using $p^{\rm CM}_{\rm total} = 0$, we find $v/c^2 = p^{\rm lab}_{\rm total}/E^{\rm lab}_{\rm total}$. This takes exactly the same form as the familiar $v/c^2 = p/E$ expression for one particle.

If we have general 3-D motion, then we can use the above reasoning with a Lorentz transformation in the x direction to show that the x component of the velocity of the CM is given by $v_x/c^2 = p^{\rm lab}_{x,\rm total}/E^{\rm lab}_{\rm total}$. Likewise for v_y and v_z. (Alternatively, if you want to, you can first transform to a frame where p_x is zero, and then transform from this frame to another one where p_y is zero (keeping p_x zero), and then finally transform from this frame to another one where p_z is zero (keeping p_x and p_y zero). You can show that the result will be a frame moving with respect to the original lab frame with the above v_x, v_y, and v_z.)

12.28. **Another perpendicular photon**

The initial energy and momentum of the system are $(5/3)mc^2 + mc^2 = (8/3)mc^2$ and $(5/3)m(4c/5) = (4/3)mc$, respectively. Conservation of energy then gives the energy of M as $E_M = (8/3)mc^2 - E$. And conservation of p_x and p_y give the components of M's momentum as $(4/3)mc$ and $-E/c$, respectively. The very important relation for M then yields (dropping the c's)

$$\begin{aligned}
\big((8/3)m - E\big)^2 &= \big((4m/3)^2 + E^2\big) + M^2 \\
\Longrightarrow \quad M &= \sqrt{(16/3)m(m - E)} \\
&\to \sqrt{(16/3)m(m - E/c^2)}.
\end{aligned} \tag{693}$$

We must therefore have $E \le mc^2$ for this setup to be possible. In the limit $E \to 0$, we have $M = 4m/\sqrt{3}$. This is just the result for a 1-D collision in which the two m's combine to form a mass M, as you can show.

12.33. **Higgs production**

(a) Let the proton mass be m and the Higgs mass be km, where $k \approx 100$ here. If the incoming proton has energy E, then the total energy and momentum are $E+m$ and $\sqrt{E^2-m^2}$, respectively. So the very important relation applied to the Higgs gives

$$(E+m)^2 = (E^2 - m^2) + (km)^2 \quad \Longrightarrow \quad E = (k^2/2 - 1)m. \tag{702}$$

The amount of energy that must be added to the rest energy of the incoming proton is therefore $\Delta E = (k^2/2 - 2)m$. Note that $\Delta E = 0$ if $k = 2$, as expected. Note also that ΔE behaves quadratically with k. If $k \approx 100$, then $\Delta E \approx 5000\, mc^2 \approx 5000$ GeV.

(b) The Higgs has zero momentum in this case, so each proton must simply have an energy $km/2$ to make a total energy of km. The amount of energy that must be added to the two rest energies is therefore $\Delta E = (k-2)m$. Again, $\Delta E = 0$ if $k = 2$, as expected. But note that ΔE now behaves linearly with k. If $k \approx 100$, then $\Delta E \approx 100\, mc^2 \approx 100$ GeV. We see that a much smaller amount of energy is required for the creation of a heavy particle if the two initial particles have equal and opposite momenta. This way the final particle has no wasted kinetic energy.