11.29. **Effectively speed c**

If L is the distance between the planets, then $L = cT$ where $T = 1$ year. The time in the planets' frame is L/v, so the time on the captain's watch is $L/\gamma v$. We therefore want

$$\frac{cT}{\gamma v} = T \quad\Longrightarrow\quad \frac{\sqrt{1-\beta^2}}{\beta} = 1 \quad\Longrightarrow\quad \beta = \frac{1}{\sqrt{2}} \quad\Longrightarrow\quad v = \frac{c}{\sqrt{2}}\,. \tag{602}$$

Alternatively, in the rocket frame, the length is L/γ, so the time is $L/\gamma v$, which agrees with above.

11.32. **Walking on a train**

(a) The train has length $4L/5$ in the ground frame. The distance it travels between the moment when its front end coincides with the near end of the tunnel and the moment when its back end coincides with the far end of the tunnel equals the sum of the lengths of the train and the tunnel, which is $4L/5 + L = 9L/5$. It covers this distance at speed $3L/5$, so the time is $3L/c$.

(b) The person moves a distance L during this time, so her speed is $c/3$.

(c) A ground observer sees the person's watch run slow by a factor $\gamma_{1/3} = 3/(2\sqrt{2})$, so the time on her watch is $(3L/c)/\gamma_{1/3} = 2\sqrt{2}L/c$.

11.42. **Tunnel fraction**

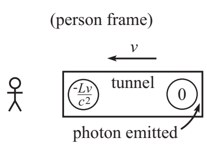

Figure 37

In the tunnel frame, things are straightforward. The sum of the distances, vt and ct, must equal L, so $t = L/(c+v)$. The person therefore travels a distance $vt = Lv/(c+v)$, which is a fraction $f = v/(c+v)$ along the tunnel.

Now consider the person's frame. First, let's assume that in the tunnel frame, clocks at the ends of the tunnel read zero when the person enters the tunnel and the photon is simultaneously emitted. Then in the train frame, the start of the process looks like the situation in Fig. 37. Because the rear clock is ahead, the photon is emitted before the tunnel reaches the person (which happens when the clock at the near end of the tunnel reads zero, which it doesn't yet).

What is the distance from the near end of the tunnel to the person at this time? The left clock must advance by Lv/c^2 by the time it reaches him. This takes a time of $\gamma(Lv/c^2)$ in the person's frame, due to time dilation. The tunnel therefore travels a distance $v(\gamma Lv/c^2)$ during this time. So this is the initial distance between the near end of the tunnel and the person.

The total distance the photon travels to reach the person is the length of the train (which is L/γ) plus the above distance, which gives

$$\frac{L}{\gamma} + \frac{\gamma L v^2}{c^2} = \gamma L\left(\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right) = \gamma L\left(\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2}{c^2}\right) = \gamma L. \tag{613}$$

(If you imagine the person holding a long ruler, you can also derive this result via a length contraction argument.) The total time of the process in the person's frame is therefore $\gamma L/c$. During this time, the tunnel travels a distance $v(\gamma L/c)$. The length of the tunnel that is beyond the person is therefore $\gamma Lv/c - \gamma Lv^2/c^2$ (that is, the distance traveled minus the initial distance from the tunnel to the person). The fraction of the tunnel that is beyond the person is therefore

$$f = \frac{\gamma L\beta - \gamma L\beta^2}{L/\gamma} = \gamma^2(\beta - \beta^2) = \frac{\beta(1-\beta)}{1-\beta^2} = \frac{\beta}{1+\beta} = \frac{v}{c+v}\,, \tag{614}$$

in agreement with the result obtained by working in the tunnel frame.

11.50. **Slanted time dilation**

Since the x speed in the original frame is zero, the transverse velocity addition formula gives the vertical speed in your frame as u/γ_v. And the horizontal speed is simply v. So the speed of the clock with respect to you is $\sqrt{v^2 + (u/\gamma_v)^2}$. The γ factor associated with this speed is (dropping the c's)

$$\gamma = \frac{1}{\sqrt{1 - v^2 - (u/\gamma_v)^2}} = \frac{1}{\sqrt{1 - v^2 - u^2(1-v^2)}} = \frac{1}{\sqrt{1-u^2}\sqrt{1-v^2}} = \gamma_u\gamma_v.$$