Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together)

Question:	1	2	Total
Points:	15	60	75
Score:			

Instructor/grader comments:

Dimensional analysis

1. **Power produced by a wind turbine**

   (a) (10 points) Use the dimensional analysis an estimate the power P produced by a wind turbine with rotor blades of length r, if the wind speed is v and the density of the air is ρ. By what factor the power increases when the wind speed doubles?

   The units of power are Watts. 1 W = 1 kg·m^2/s^3.

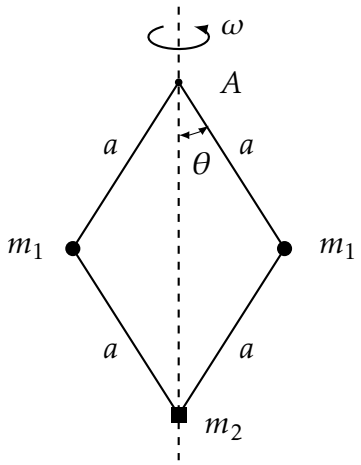
   (b) (5 points) Suppose r = 100 m and the wind speed is v = 10 m/s. Give the numerical estimate of the power generated by the turbine. Assume that $\rho \approx$ 1kg/m^3.

Centrifugal governors were invented by Christiaan Huygens and used to regulate the distance and pressure between millstones in windmills in the 17th century. In 1788, James Watt adapted a centrifugal governors to control his steam engine where it regulates the admission of steam into the cylinder(s), a development that proved so important he is sometimes called the inventor. Centrifugal governors' widest use was on steam engines during the *Steam Age* in the 19th century. (From Wikipedia.)

2. Consider an oversimplified centrifugal governor shown in Fig. 1. The frictionless system placed in a uniform gravitational field (acceleration g). The masses are connected by massless rods of length a. The mass m_2 slides along a vertical axis and the whole system rotates about this axis with a constant angular speed ω.

   Use the angle θ (see Fig. 1) as the generalized coordinate that describes the configuration of the system.

Figure 1:



(a) (10 points) What are the kinetic and potential energy of mass m_2: $T_2\left(\theta, \dot{\theta}\right)$, $V_2(\theta)$?

(b) (10 points) What are the kinetic and potential energy of mass m_1: $T_1\left(\theta, \dot{\theta}\right)$, $V_2(\theta)$?

(c) (15 points) To simplify the algebraic expressions, consider the case $m_1 = m_2 = m$. Introduce the characteristic frequency of free oscillations, $\omega_0^2 = g/a$.

Write down the Lagrangian of the system. Find the conserved energy integral, E, of the system. Is the energy integral equal to the sum of kinetic and potential energy of the system? Explain why or why not.

(d) (15 points) Write down the equation of motion of the system.

Hint: Note that $\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{\theta}\sin^2\theta\right) \neq \ddot{\theta}\sin^2\theta$

(e) (10 points) From the equation of motion determine the equilibrium configuration of the governor, $\theta_e(\omega)$. What should the angular speed ω be such that $\theta_e = \pi/4$?

Hint: equilibrium configuration is the solution of the equation of motion such that $\dot{\theta} = 0$, $\ddot{\theta} = 0$, $\theta \neq 0$.