Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	20	10	20	20	70
Score:					

Instructor/grader comments:

Dimensional analysis.

1. (a) (10 points) Estimate the torque, τ ($[\tau] = M L^2 T^{-2}$), created by viscous friction forces acting on a large disk of radius R that rotates with constant angular frequency ω in a fluid with kinematic viscosity ν ($[\nu] = L^2 T^{-1}$) and density ρ. The experiment shows that $\tau \sim \sqrt{\nu}$.

   (b) (5 points) How the torque changes if you quadruple the rotation frequency?

   (c) (5 points) Estimate the force one needs to apply tangentially at the edge of a disk of radius 1 m immersed into water so that the disk rotates with the frequency $\omega =$ 4 radian/sec. For the purpose of the present problem use $\nu_{\text{water}} = 10^{-6}$ m^2/sec.

2. (10 points) **Damped harmonic motion**

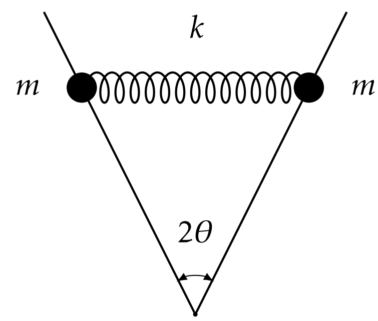
   A *critically* damped oscillator with natural frequency ω starts out at $t = 0$ at the position $x_0 > 0$. What is the maximum initial speed v_0 (directed toward the origin) it can have and not cross the origin?

   Hint: recall that for a critically damped oscillator, $x(t) = (A + Bt)e^{-\omega t}$, where A and B are determined by the initial conditions. Find A and B in terms of x_0 and v_0. Next, make sure that 'crossing the origin' equation $x(t_c) = 0$ has no solutions for $t > 0$.

3. (20 points) Two particles are constrained to move along two horizontal frictionless rails that make an angle 2θ with respect to each other. The particles are connected by a spring with elastic constant k, whose relaxed length is at the position shown in Fig. 1. What is the frequency of oscillations for the motion where the spring remains parallel to the position shown? Verify that your solution make sense in the limits $\theta \to 0$ (i.e. parallel rails) and $\theta \to \frac{\pi}{2}$ (single rail case).

   Hints: use the symmetry of the system and consider the motion of one of the particles. Find the equation of motion for the displacement of the mass along the rail measured from the equilibrium position. Remember to take into account only the component of the spring force directed along the rail.

Figure 1:



4. (20 points) Two horizontal frictionless rails make an angle α with each other, as shown in Fig. 2. Each rail has a bead of mass m on it, and the beads are connected by a spring with spring constant k and relaxed length zero. Assume that one of the rails is positioned a tiny distance above the other, so the beads can pass freely through the crossing.

   Use the symmetry of the system and sketch the normal modes. Find the frequencies of normal modes.

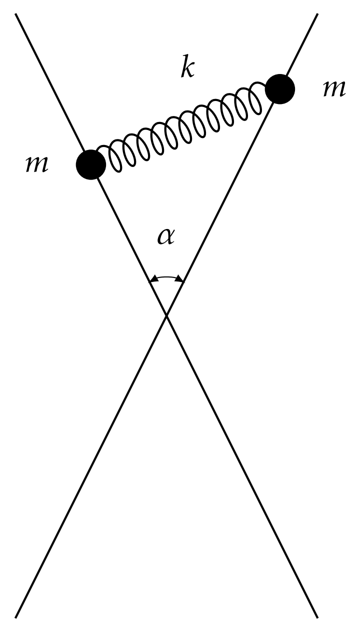


Figure 2: