Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	20	10	25	55
Score:				

Instructor/grader comments:

Dimensional analysis.

1. *White dwarfs* are thought to be the final evolutionary state of stars whose mass is not high enough to become neutron stars.

   After the hydrogen–fusing lifetime of a star of low or medium mass ends, it will expand to a red giant which fuses helium to carbon and oxygen. If a red giant has insufficient mass to generate the core temperatures required to fuse carbon, which is around 10^9 K, an inert mass of carbon and oxygen will build up at its center. After shedding its outer layers to form a planetary nebula, the star will leave behind the core, which forms the white dwarf.

   Quantum mechanics predicts that there is a relation between the radius of a white dwarf and its mass, $R = R(M)$. Explain briefly but clearly why neither of the formulas below can be the correct expressions for for $R(M)$. In those formulas m is electron mass, N is the number of electrons per unit mass ($[\mathrm{N}] = \mathrm{M}^{-1}$), $\hbar$ is the so called Plank constant, $\hbar = 1.054 \times 10^{-34}\ \mathrm{m}^2\mathrm{kg/s}$, G is the gravitational constant.

   (a) (5 points) What is the dimension of gravitational constant G? Use the Newton's law of gravity and show your derivation.

   $$F = G\,\frac{m_1\, m_2}{r^2}.$$

   (b) (5 points) $R \approx \dfrac{N^{5/3}\hbar^2}{2mGM^{2/3}}$

   (c) (5 points) $R \approx \dfrac{N^{5/3}\hbar^2}{2(m+G)M^{1/3}}$

   (d) (5 points) $R \approx \dfrac{N^{5/3}\hbar^2}{2mGM^{1/3}} \ln\dfrac{\hbar}{N}$

   Provide just one reason for each answer.

2. (10 points) **Drag force in an ideal fluid**

   A sphere is executing an oscillatory motion in an ideal (i.e. incompressible and inviscid) fluid under the action of an external force. Use the dimensional analysis to estimate the drag force acting on the sphere. Assume that the relevant physical parameters are: the radius of the sphere, R; the density of the fluid, ρ; the acceleration of the sphere, a.

   Comment: if derived correctly, your result predicts that the drag force is zero on a body moving in incompressible and inviscid fluid with constant velocity. This (correct) result is known as *d'Alembert's paradox*. It is called a paradox because when the result was discovered in 1749, it was (wrongly) assumed that it indicates the flaws in the fluid dynamics.

3. (25 points) Consider the Atwood's machine in Fig. 1, with masses m, $2m$, and $3m$. Find the accelerations of the masses and the tension(s) in the string.

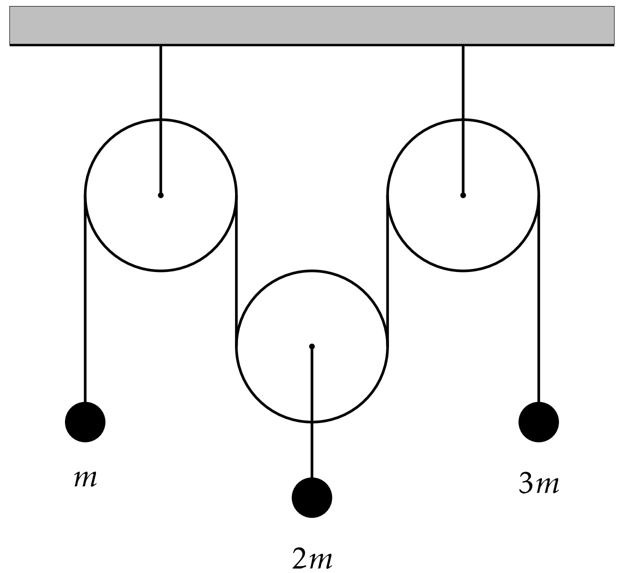

Figure 1: Atwood's machine