Name: ____________________________

Date: ____________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	5	Total
Points:	10	20	15	15	35	95
Score:						

Instructor/grader comments:

1. (10 points)

   I watched the video R. Feynman, The Character of Physical Law, Lecture 2, *The relation of Mathematics and Physics* assigned as a part of HW1.

   Sign and date here: ____________________________________

Dimensional analysis. Limiting cases.

2. **Entropy of mixing**

   In statistical mechanics, the *dimensionless entropy* is a property of a physical system that is closely related to the number of possible microscopic configurations of the system.

   For the purpose of the present problem you need to know that (a) the dimensionless entropy is just a number, and (b) the entropy of a an isolated system is always increasing when the system evolves spontaneously toward an equilibrium.

   Let's consider two different gases with numbers of particles N_1 and N_2 in rigid containers of volumes V_1 and V_2 at the same temperature and pressure. The containers then are connected and the gases spontaneously mixed. The volume of the mixture becomes $V_1 + V_2$; the pressure and temperature obviously remain the same. The entropy, however, changes:

   $$\Delta S = S_{\text{mixed}} - S_{\text{separated}} \neq 0 \tag{1}$$

   The quantity ΔS is called the *entropy of mixing*. This quantity is positive, i.e. the entropy increases on mixing, as it should, in accordance with the fact that some work must be done in order to separate again the molecules of the two gases.

   Explain briefly but clearly why neither of the formulas below can be the correct expressions for the entropy of mixing of two gases. Use only dimensional analysis and/or consider suitable limiting cases.

   (a) (5 points) $\Delta S = N_1 \ln V_1 + N_2 \ln V_2$

   (b) (5 points) $\Delta S = N_1 N_2 \ln \frac{V_1}{V_2}$

(c) (5 points) $\Delta S = (N_1 + N_2)\sqrt{\frac{V_1+V_2}{V_1-V2}}$

(d) (5 points) $\Delta S = V_1^{N_1} + V_2^{N_2}$

3. **The problem of a strong explosion**

The so-called *shock waves* are formed during strong explosions in gases. The shock wave in a uniform gas is a sphere with the radius that increases with time.

(a) (10 points) Use the dimensional analysis to obtain the formula for the radius, R, of the shock wave created by a strong explosion. The relevant physical parameters for this problem are: the time elapsed since the instant of the explosion, t; the density of the surrounding gas, ρ; and the energy released by the explosion, E ($[E] = J = ML^2T^{-2}$).

(b) (5 points) Use the information in Fig. 1 for t and R to estimate the energy released during the first atomic bomb test in 1945. For the purpose of the present problem use $\rho \approx 1\ \mathrm{kg/m^3}$.

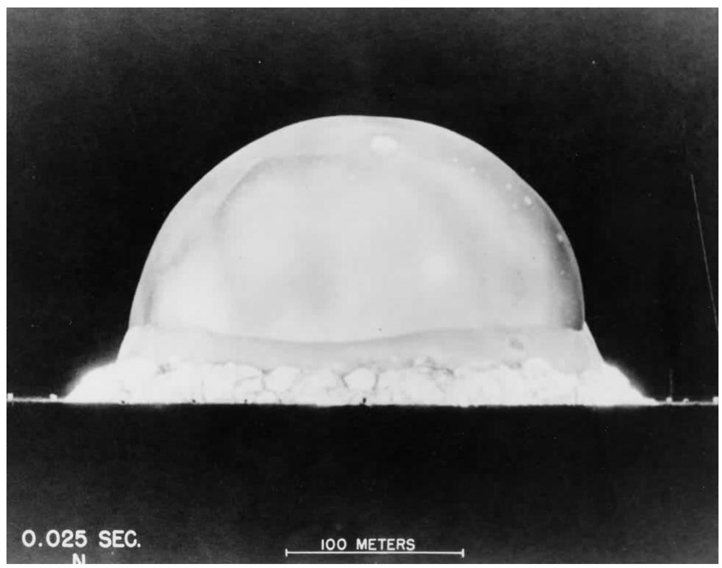

Figure 1: A photo of a shock wave formed during the first nuclear explosion test occurred on July 16, 1945.

Projectile motion

4. (15 points) At what angle, θ, should a ball be thrown so that its maximum height equals the horizontal distance traveled? Obtain an expression for $\tan\theta$ and find θ numerically.

5. A projectile is fired in a giant hall on spacecraft *Discovery Nine*. A projectile is fired with speed v_0 at an angle θ above the horizontal. The acceleration due to the artificial gravity increases linearly with time, starting with a value of zero when the projectile is fired. In other words, the acceleration of artificial gravity $g(t) = \beta t$, where β is a given constant.

   (a) (5 points) What is the dimension of the parameter β?

   (b) (15 points) What horizontal distance, s, does the projectile travel?

   (c) (10 points) Use dimensional analysis and consider limiting cases $v_0 \to 0$, $\theta \to \pi/2$, $\theta \to 0$, and $\beta \to 0$ to verify your expression for s.

   (d) (5 points) What should θ be to maximize the horizontal distance of the projectile? Obtain an expression for $\tan\theta$ and find θ numerically.