Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	25	15	40	80
Score:				

Instructor/grader comments:

Dimensional analysis. Limiting cases.

1. A truncated pyramid with a regular hexagon with side length a as the bottom base, a regular hexagon with side length b as the top base, and the height h is shown in Fig. 1.

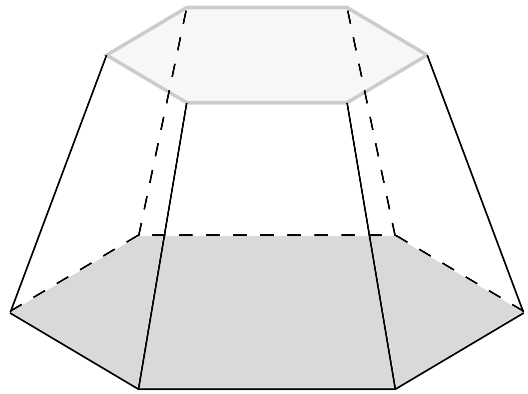

Figure 1: Truncated pyramid.

Explain briefly but clearly[1] why neither of the formulas below can be the correct expressions for for the volume of this pyramid:

(a) (5 points) $V = a\,b\,h$

(b) (5 points) $V = a\,h + b^2$

(c) (5 points) $V = 2h(a^2 + b^2)$

(d) (5 points) $V = \log(h^3 + a^3 + b^3)$

(e) (5 points) $V = h^3\,\frac{a}{b}$

Provide just one reason for each answer.

Recall that the area of a regular hexagon is less than the area of its circumscribed circle (which is πa^2).

[1] Examples of possible answers – just to give an idea regarding the style of expected answers – most examples are wrong for this particular problem: "wrong dimension [L^2] instead of correct [$M \cdot L$]"; "wrong limit $T \to 0$ as $s \to \infty$, must be $T \to \infty$ as $s \to \infty$"; "correct answer cannot contain a transcendental function of a dimensional argument".

2. (15 points) Basilisk lizards, a group of reptiles found in Central America, are capable of running across water using only their feet as a source of both lift and thrust. The basilisk's weight is too great to be supported by the water's surface tension, so animals cannot simply reside at rest on the water surface, but must be in a constant state of motion.

   Use the dimensional analysis to estimate the vertical speed (in m/s) of the lizard's foot required for running across water. Assume that the relevant physical parameters are the weight of the lizard, W, the area of its foot, A, and the density of the water, ρ. For your numerical estimate use the following values: $W = 0.9$ N, $A = 10^{-4}$ m^2, $\rho = 10^3$ kg/m^3.

Integration of the equations of motion

3. Consider the vertical motion of a skydiver of mass m falling through air. The two forces acting on the diver are the force of gravity, $F_g = mg$, and the drag force,

$$F_d(v) = -\beta v^2.$$

The initial vertical speed of the diver is $v_0 = 0$.

(a) (5 points) Write the equation of motion in vertical direction

(b) (5 points) What is the dimension of the parameter β? Show your derivation.

(c) (5 points) Find the terminal speed of the skydiver, v_t.

(d) (10 points) Integrate the equation of motion. Find the speed of the skydiver vs time, $v(t)$.

Hint: You may need to use the value of the following integral:

$$\int \frac{\mathrm{d}u}{1-u^2} = \mathrm{arctahn}(u).$$

(e) (10 points) Verify that your result for $v(t)$ make sense by considering the limits (a) $t \to 0$, and (b) $t \to \infty$.

Hint: You may need to use the following approximations: $\tanh(\varepsilon) \approx \varepsilon$ when $\varepsilon \ll 1$, and $\tanh(\varepsilon) \approx 1$ which when $\varepsilon \gg 1$.

(f) (5 points) Estimate the terminal velocity of a skydiver of mass $m = 64$ kg and $\beta = 0.25$ in the units you determined in Part (b). For the purpose of this estimation assume that $g = 9\ \mathrm{m/s^2}$.