Question:	1	2	3	Total
Points:	30	40	30	100

Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form.

Dimensional analysis

1. **The mass and the volume of the pendant drop**

   A drop of liquid hangs from the tip of a vertical capillary tube, for example a pipette fitted with a rubber bulb, through which liquid can be slowly injected to feed the drop. If the feeding rate is slow enough, the drop grows through a sequence of equilibrium shapes with continuously increasing volume until it becomes unstable and detaches.

   (a) (10 points) Use the dimensional analysis an estimate the mass of a drop detached from a pipette of diameter a. The other relevant parameters are the acceleration of gravity g, and the surface tension α. (α is measured in units of force per length.)

   (b) (5 points) Using your expression for the mass of the drop, write the expression for its volume.

   (c) (5 points) Give the numerical estimate for the volume, V, of a drop of water created using a pipette of diameter of 2 mm. For the purpose of this exercise assume that $g \approx 10\, m/s^2$ and $\alpha \approx 0.07\, N/m$.

   (d) (10 points) Would you be able to obtain the formula for the volume of the drop directly using the dimensionless analysis? To answer this question, write the equations that determine the powers of variables in the formula for the volume (you need to add the density of fluid to the list of the variables) and see if you have enough equations to determine all parameters.

Small oscillations

2. A particle of mass m moves along a straight line under the influence of force with the potential

$$V(x) = \frac{A}{x^2} - \frac{B}{x},$$

   where $A, B > 0$.

   (a) (5 points) What is the dimensions of parameters A and B?

   (b) (5 points) Sketch the potential $V(x)$

(c) (5 points) Find the force, $F(x)$, acting on the particle. Verify the dimension of your expression.

(d) (5 points) What is the equilibrium position of the particle, x_0? Verify the dimension of your expression.

(e) (10 points) Find the frequency of small oscillations, ω, around the equilibrium point. Verify the dimension of your expression.

(f) (10 points) What happens to x_0 and ω as the parameter $B \to 0$? Interpret your result.

Conservation of energy

3. A frictionless tube is bent into the shape of a semicircle with radius R. The semicircle is tilted so that its diameter makes a fixed angle θ with respect to the vertical, as shown in Fig. 1. A small mass is released from rest at the top of the tube and slides down through it. When the mass leaves the tube, it undergoes projectile motion.

Figure 1:

(a) (10 points) What is the speed of the mass and the angle its velocity makes with the horizontal direction at the instant when the mass leaves the tube?

(b) (10 points) Find $d(\theta)$ – the distance traveled in the projectile motion, up to the time when the mass returns to the height it had when it left the tube. Assume that you know everything you may need about the projectile motion.

(c) (10 points) What should θ be so that d is as large as possible? What is this largest d?