Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	Total
Points:	85	85
Score:		

Instructor/grader comments:

From Wikipedia, the free encyclopedia:

> *Centrifugal governors* were invented by Christiaan Huygens and used to regulate the distance and pressure between millstones in windmills in the 17th century. In 1788, James Watt adapted one to control his steam engine where it regulates the admission of steam into the cylinder(s), a development that proved so important he is sometimes called the inventor. Centrifugal governors' widest use was on steam engines during the *Steam Age* in the 19th century.

1. Consider an oversimplified centrifugal governor shown in Fig. 1. The frictionless system placed in a uniform gravitational field (acceleration g). The masses are connected by massless rods of length l. The mass m_2 slides along a vertical axis and the whole system rotates about this axis with a constant angular speed Ω.

   Hint: Use the angle θ as the generalized coordinate that describes the configuration of the system.

   (a) (10 points) What is the kinetic and the potential energy of mass m_2?

   (b) (15 points) What is the kinetic and the potential energy of mass m_1?

   (c) (10 points) Write down the Lagrangian of the system.

   (d) (15 points) Find the generalized momentum of the system and the conserved energy integral. Is the energy integral equal to the sum of kinetic and potential energy of the system? Explain why or why not.

   (e) (15 points) Write down the equation of motion. (Be careful when calculating full derivative with respect to time.)

   (f) (10 points) Simplify the equation of motion assuming that $\dot{\theta}$ and $\ddot{\theta}$ (but not θ itself) are very small.

   (g) (10 points) From the simplified equation of motion determine what should be the rotation speed Ω such that the equilibrium value of θ be $\pi/2$.

Figure 1: