Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	25	25	35	85
Score:				

Instructor/grader comments:

Lagrangian mechanics

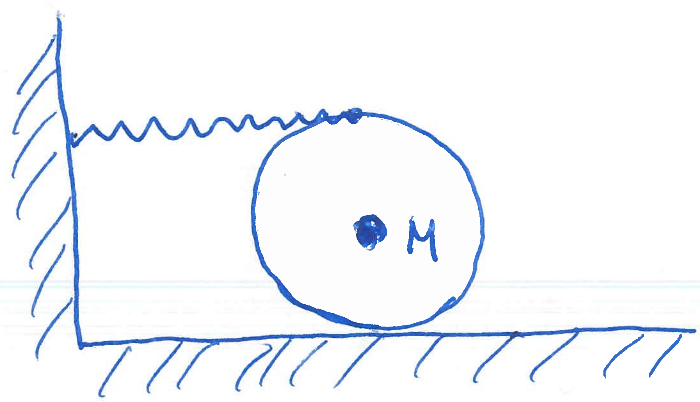
1. The top of a wheel of mass M and radius R is connected to a spring (at its equilibrium length) with spring constant k, as shown in Fig. 1. All the mass of the wheel is at its center. The wheel rolls without slipping.

   (a) (5 points) What is a suitable generalized coordinate that describes the configuration of the system? (There are several equally good choices.)

   (b) (10 points) Write down the Lagrangian of the system in terms of your generalized coordinate. (Assume that the linear amplitude of the rolling motion is much smaller than the radius of the wheel and the equilibrium length of the spring.)

   (c) (10 points) What is the frequency of small oscillations of the wheel?

Figure 1:

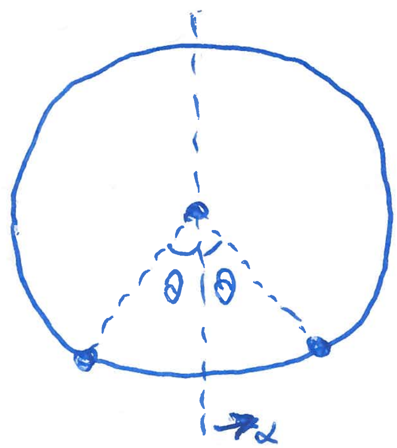


2. Two equal masses are attached to a massless hoop of radius R that is free to rotate about its center in a vertical plane. The angle between the masses is 2θ, as shown in Fig. 2. Let α be the angle of rotation of the hoop measured from its equilibrium.

   (a) (15 points) What is the Lagrangian of the system? in terms of α ?

   (b) (10 points) What is the frequency of small oscillations of the hoop?

Figure 2:



3. A bead is free to slide along a frictionless hoop of radius R. The hoop rotates with the constant angular speed ω around a vertical diameter (see Fig. 3).

   (a) (15 points) What is the Lagrangian of the system in terms of θ?

   (b) (5 points) Write down the equation of motion of the bead.

   (c) (15 points) What is the smallest angular speed such that the equilibrium position of the bead is not at the bottom of the hoop?

Figure 3: