Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	15	35	20	20	90
Score:					

Instructor/grader comments:

Dimensional analysis

1. Suppose you drill a tunnel through the Earth's center. Then you drop an object into the tunnel.

   (a) (2 points) What is the dimension of gravitational constant G? Use the Newton's law of gravity $F = G\frac{m_1 m_2}{r^2}$ and show your derivation.

   (b) (8 points) Use the dimensional analysis to find the time, T, required for the object's passage from one side of the Earth to the other. Assume that the Earth is a uniform sphere. Neglect the Earth's rotation and the resistance of the air in the tunnel. T possibly depends on the gravitational constant G, the (average) density of the Earth ρ, and the Earth's radius R.

   (c) (2 points) Estimate the passage time, in minutes, for Earth: $\rho \approx 5.51 \times 10^3\,\mathrm{kg/m^3}$, $G = 6.67 \times 10^{-11}$ SI units. For the purpose of this question assume $1\,\mathrm{minute} \approx 50\,\mathrm{sec}$.

   (d) (3 points) How the passage time is going to change if you repeat the experiment on a planet twice the radius of the Earth, all other parameters being the same?

Conservation of energy

2. Two beads of mass m are initially at rest at the top of a frictionless hoop of mass M and radius R, which stands vertically on the ground. The beads are given identical tiny kicks, and they slide down the hoop, one to the right and one to the left, as shown in Fig. 1.

   (a) (5 points) What is the speed of the beads, $v(\theta)$ as a function of angle θ?

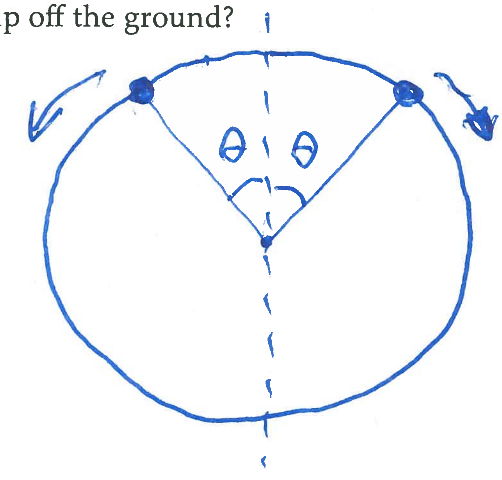
   (b) (5 points) What is the normal force, $N(\theta)$, acting on each bead from the hoop?

   (c) (10 points) What is the net force, $F(\theta)$, acting on the hoop from the beads? For what values of θ this force is directed upward?

   (d) (10 points) What is the maximal value of the upward force? What is the corresponding value of θ?

   (e) (5 points) What is the smallest value of M (measured in units of m) for which the hoop never rises up off the ground?

Figure 1:



3. (20 points) A pendulum of length L is held with its string horizontal, and then released. The string runs into a peg a distance d below the pivot, as shown in Fig. 2. What is the smallest value of d for which the string remains taut at all times?

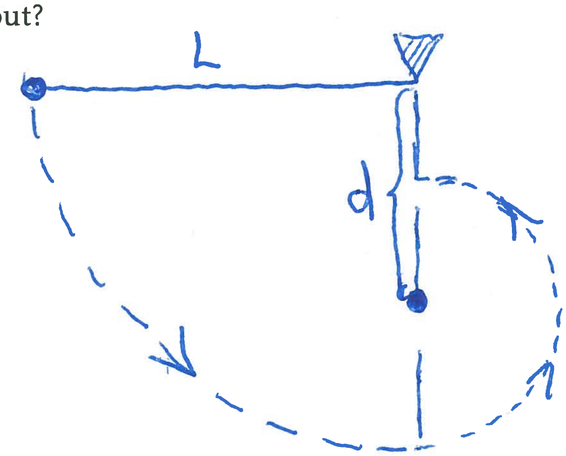
Hints:

(a) What is the speed of the mass at the end of the pendulum when the string first touches the peg?

(b) What is the speed of the mass when it reaches the highest point when rotating around the peg with the string remaining taut? (Assume that the speed is large enough so that the pendulum actually reaches the highest possible point.)

(c) What is the smallest speed that the pendulum must have at the highest point to keep the string tout?

Figure 2:



4. A particle of mass m and charge q is constrained to move along a straight line between two point charges of equal charge Q separated by distance $2L$. The sign of all three charges is the same so that the charge q is repelled by the other two particles.

   (a) (5 points) What is equilibrium position of charge q? Use the symmetry of the system. Is the equilibrium stable or unstable?

   (b) (5 points) Chose the origin of your reference frame at the equilibrium position of charge q. What is the potential energy of the charge as a function of its displacement from the equilibrium, x? Verify that $x = 0$ is indeed the equilibrium position of the charge q.

   (c) (5 points) What is the frequency of small oscillations, ω, of the charge around the equilibrium?

   (d) (5 points) Estimate ω if $q = Q = e$, where e is the charge of the electron, m is the mass of the electron, and $L = 10^{-10}$ m.