Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	10	10	20	30	70
Score:					

Instructor/grader comments:

Dimensional analysis.

1. (10 points) Estimate the torque, τ ($[\tau] = M L^2 T^{-2}$), created by viscous friction forces acting on a large disk of radius R that rotates in a fluid with kinematic viscosity ν ($[\nu] = L^2 T^{-1}$) and density ρ with constant angular frequency Ω. The experiment shows that $\tau \sim \nu^{\frac{1}{2}}$.

   How the torque changes if you quadruple the rotation frequency?

   Estimate the torque for a disk of radius $1\,m$ rotating in water with the frequency 1 radian/sec. $\nu_{\text{water}} = 10^{-6}\ m^2/sec.$

   Hint: all physical parameters mentioned in the problem statement are relevant.

2. (10 points) **Damped harmonic motion**

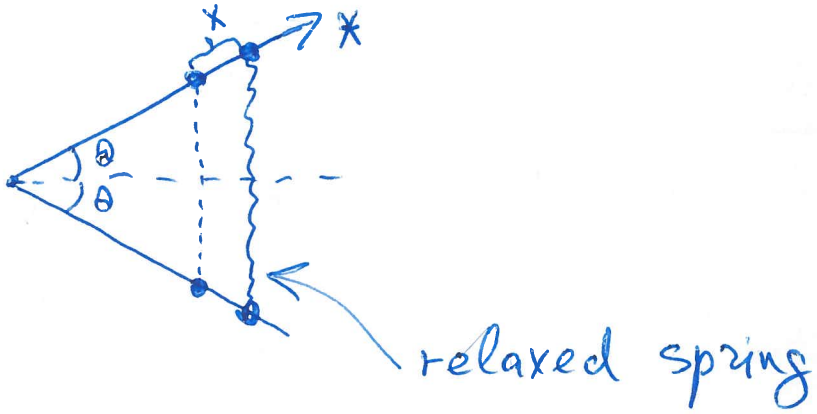
A *critically* damped oscillator with natural frequency ω starts out at $t = 0$ at the position $x_0 > 0$. What is the maximum initial speed v_0 (directed toward the origin) it can have and not cross the origin.

Hints: recall that for a critically damped oscillator $x(t) = (A + Bt)e^{-\omega t}$, where A and B are determined by the initial conditions. Find A and B corresponding to the conditions of the problem. Next, make sure that 'crossing the origin' equation $x(t_c) = 0$ has no solutions $t > 0$.

3. (20 points) Two particles are constrained to move along two horizontal frictionless rails that make an angle 2θ with respect to each other. They are connected by a spring with elastic constant k, whose relaxed length is as show in Fig. 1. What is the frequency of oscillations for the motion where the spring remains parallel to the position shown?

   Hint: use the displacement along the rail from the equilibrium position as your dependent variable.

Figure 1:



4. (30 points) Two particles are constrained to move along two horizontal frictionless rails that make an angle α with respect to each other, as shown in Fig. 2. They are connected by a spring with elastic constant k and relaxed length zero. Find the normal modes. Assume that one of the rails is positioned a tiny distance above the other, so particles can pass freely through the crossing.

Figure 2: