Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	10	25	20	20	75
Score:					

Instructor/grader comments:

Dimensional analysis.

1. (10 points) Use the dimensional analysis to estimate the speed of a satellite whose orbit is just above the earth's surface. Assume that the relevant physical parameters are the acceleration of gravity at the earth's surface, g, and the radius of the orbit, R. Give the numerical estimate. For the purpose of this problem assume that $g = 10\ \mathrm{m/s^2}$ and $R = 6400$ km.

Solving differential equations of motion

2. A particle of mass m is subject to a force

$$F(v) = -\beta v^2.$$

The initial position, of the particle is $x(0) = 0$. The initial speed is v_0.

(a) (5 points) What is the dimension of the parameter β. Show your derivation.

(b) (10 points) Find the speed of the particle vs time, $v(t)$.

(c) (5 points) Find the position of the particle vs time, $x(t)$.

(d) (5 points) Verify that your results make sense by considering the limits (a) $\beta \to 0$, m is finite, and (b) $m \to \infty$, β is finite.

Hint: You may need to use the following approximation: $\ln(1+\epsilon) \approx \epsilon$ valid when $\epsilon \ll 1$.

3. A particle of mass m is subject to a force

$$F(x) = \kappa x,$$

where $\kappa > 0$. The initial position, of the particle is x_0. The initial speed is zero.

(a) (5 points) Find the speed of the particle vs the position, $v(x)$.

(b) (10 points) Find the position of the particle vs time, $x(t)$.

Hint: To evaluate the required integral you may need to use the substitution $x = x_0 \cosh(u)$. For the reference, $\cosh^2 u - 1 = \sinh^2 u$, $\mathrm{d}(\cosh u) = \sinh u \, \mathrm{d}u$.

(c) (5 points) Verify that your results make sense by considering the limits (a) $\kappa \to 0$, m is finite, and (b) $m \to \infty$, κ is finite.

Projectile motion

4. (20 points) A cart is held at rest on a frictionless inclined plane. A tube is positioned in the cart with its axis perpendicular to the plane. The cart is released, and at some later time a ball is fired from the tube. Show that the ball eventually lands back in the tube.

   Hint: the problem requires practically no calculations if the coordinate system is chosen wisely.