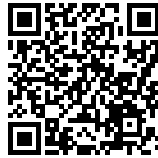

A FLY AND A DROP OF HONEY

MECHANICS I, SPRING SEMESTER 2019

http://www.phys.uconn.edu/~rozman/Courses/P3101_19S/



Last modified: February 6, 2019

A smart fly noticed a drop of honey on the table, while flying horizontally exactly above the drop with the speed v_0 at the height H . The fly is able to move with the acceleration not more than a in any direction. Find the least time required for the fly to get to the honey, t_r . Find the parametric equation of the 'optimal' trajectory of the fly.

$$\sqrt{(v_0 t_r)^2 + H^2} = \frac{a t_r^2}{2}. \quad (1)$$

$$\frac{a^2}{4} t_r^4 - v_0^2 t_r^2 - H^2 = 0. \quad (2)$$

$$t_r^4 - \frac{4v_0^2}{a^2} t_r^2 - \frac{4H^2}{a^2} = 0. \quad (3)$$

$$t_r^2 = \frac{2v_0^2}{a^2} + \sqrt{\frac{4v_0^4}{a^4} + \frac{4H^2}{a^2}} = \frac{2v_0^2}{a^2} + \frac{2v_0^2}{a^2} \sqrt{1 + \frac{H^2 a^2}{v_0^4}} = \frac{2v_0^2}{a^2} \left(1 + \sqrt{1 + \frac{H^2 a^2}{v_0^4}} \right). \quad (4)$$

$$\kappa = \frac{H a}{v_0^2} \quad (5)$$

$$t_r^2 = \frac{2v_0^2}{a^2} (1 + \sqrt{1 + \kappa^2}). \quad (6)$$

$$t_r = \frac{\sqrt{2}v_0}{a} \sqrt{1 + \sqrt{1 + \kappa^2}} \quad (7)$$

$$S = \frac{at_r^2}{2} = \frac{v_0^2}{a} (1 + \sqrt{1 + \kappa^2}) = H \left(\frac{v_0^2}{Ha} \right) (1 + \sqrt{1 + \kappa^2}) = \frac{H}{\kappa} (1 + \sqrt{1 + \kappa^2}). \quad (8)$$

$$L = v_0 t_r = \frac{\sqrt{2}v_0^2}{a} \sqrt{1 + \sqrt{1 + \kappa^2}} = \sqrt{2}H \left(\frac{v_0^2}{Ha} \right) \sqrt{1 + \sqrt{1 + \kappa^2}} = \sqrt{2} \frac{H}{\kappa} \sqrt{1 + \sqrt{1 + \kappa^2}}. \quad (9)$$

$$\sin \alpha = \frac{L}{S} = \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 + \kappa^2}}} \quad (10)$$

$$\cos \alpha = \frac{H}{S} = \frac{\kappa}{1 + \sqrt{1 + \kappa^2}} \quad (11)$$

$$x(t) = -\frac{at^2}{2} \sin \alpha, \quad (12)$$

$$y(t) = -\frac{at^2}{2} \cos \alpha. \quad (13)$$

$$\tau = \frac{t}{t_r}, \quad t = t_r \tau, \quad 0 \leq \tau \leq 1. \quad (14)$$

$$\frac{y(\tau)}{H} = -\frac{a}{2H} t_r^2 \tau^2 \cos \alpha = -\frac{a}{2H} \frac{2v_0^2}{a^2} \tau^2 (1 + \sqrt{1 + \kappa^2}) \frac{\kappa}{1 + \sqrt{1 + \kappa^2}} = \tau^2 \quad (15)$$

$$\frac{x(\tau)}{H} = -\frac{a}{2H} t_r^2 \tau^2 \sin \alpha = -\frac{a}{2H} \frac{2v_0^2}{a^2} \tau^2 (1 + \sqrt{1 + \kappa^2}) \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 + \kappa^2}}} \quad (16)$$

$$= -\frac{\sqrt{2}\sqrt{1 + \sqrt{1 + \kappa^2}}}{\kappa} \tau^2 \quad (17)$$