A FLY AND A DROP OF HONEY

MECHANICS I, SPRING SEMESTER 2019

http://www.phys.uconn.edu/~rozman/Courses/P3101_19S/

Last modified: February 6, 2019

A smart fly noticed a drop of honey on the table, while flying horizontally exactly above the drop with the speed v_0 at the height H. The fly is able to move with the acceleration not more than a in any direction. Find the least time required for the fly to get to the honey, t_r. Find the parametric equation of the 'optimal' trajectory of the fly.

$$\sqrt{(v_0\,t_r)^2 + H^2} = \frac{a\,t_r^2}{2}. \tag{1}$$

$$\frac{a^2}{4}\,t_r^4 - v_0^2\,t_r^2 - H^2 = 0. \tag{2}$$

$$t_r^4 - \frac{4\,v_0^2}{a^2}\,t_r^2 - \frac{4\,H^2}{a^2} = 0. \tag{3}$$

$$t_r^2 = \frac{2\,v_0^2}{a^2} + \sqrt{\frac{4\,v_0^4}{a^4} + \frac{4\,H^2}{a^2}} = \frac{2\,v_0^2}{a^2} + \frac{2\,v_0^2}{a^2}\sqrt{1 + \frac{H^2\,a^2}{v_0^4}} = \frac{2\,v_0^2}{a^2}\left(1 + \sqrt{1 + \frac{H^2\,a^2}{v_0^4}}\right). \tag{4}$$

$$\kappa = \frac{H\,a}{v_0^2} \tag{5}$$

$$t_r^2 = \frac{2\,v_0^2}{a^2}\left(1+\sqrt{1+\kappa^2}\right). \tag{6}$$

$$t_r = \frac{\sqrt{2}\,v_0}{a}\sqrt{1+\sqrt{1+\kappa^2}} \tag{7}$$

$$S = \frac{a\,t_r^2}{2} = \frac{v_0^2}{a}\left(1+\sqrt{1+\kappa^2}\right) = H\left(\frac{v_0^2}{H\,a}\right)\left(1+\sqrt{1+\kappa^2}\right) = \frac{H}{\kappa}\left(1+\sqrt{1+\kappa^2}\right). \tag{8}$$

$$L = v_0\,t_r = \frac{\sqrt{2}\,v_0^2}{a}\sqrt{1+\sqrt{1+\kappa^2}} = \sqrt{2}\,H\left(\frac{v_0^2}{H\,a}\right)\sqrt{1+\sqrt{1+\kappa^2}} = \sqrt{2}\,\frac{H}{\kappa}\sqrt{1+\sqrt{1+\kappa^2}}. \tag{9}$$

$$\sin\alpha = \frac{L}{S} = \frac{\sqrt{2}}{\sqrt{1+\sqrt{1+\kappa^2}}} \tag{10}$$

$$\cos\alpha = \frac{H}{S} = \frac{\kappa}{1+\sqrt{1+\kappa^2}} \tag{11}$$

$$x(t) = -\frac{a\,t^2}{2}\sin\alpha, \tag{12}$$

$$y(t) = -\frac{a\,t^2}{2}\cos\alpha. \tag{13}$$

$$\tau = \frac{t}{t_r}, \quad t = t_r\,\tau, \quad 0 \le \tau \le 1. \tag{14}$$

$$\frac{y(\tau)}{H} = -\frac{a}{2H}\,t_r^2\,\tau^2\cos\alpha = -\frac{a}{2H}\,\frac{2\,v_0^2}{a^2}\,\tau^2\left(1+\sqrt{1+\kappa^2}\right)\frac{\kappa}{1+\sqrt{1+\kappa^2}} = \tau^2 \tag{15}$$

$$\frac{x(\tau)}{H} = -\frac{a}{2H}\,t_r^2\,\tau^2\sin\alpha = -\frac{a}{2H}\,\frac{2\,v_0^2}{a^2}\,\tau^2\left(1+\sqrt{1+\kappa^2}\right)\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+\kappa^2}}} \tag{16}$$

$$= -\frac{\sqrt{2}\sqrt{1+\sqrt{1+\kappa^2}}}{\kappa}\tau^2 \tag{17}$$