3.28. **Atwood's 2**

By conservation of string, the downward acceleration of the left mass is 3 times the upward acceleration of the right mass, because three segments of string are each shortened by d if the right mass rises by d. Also, three tensions pull up on the right mass. So the $F = ma$ equations for the left and right masses are, respectively,

$$mg - T = ma_1, \qquad 3T - mg = ma_2. \tag{21}$$

Solving these, along with $a_1 = 3a_2$, gives $a_1 = 3g/5$ downward, and $a_2 = g/5$ upward.

3.52. **Increasing distance**

(a) Since $x = (v\cos\theta)t$ and $y = (v\sin\theta)t - gt^2/2$, the square of the distance from you is

$$\ell^2 = x^2 + y^2 = (v\cos\theta\, t)^2 + (v\sin\theta\, t - gt^2/2)^2 = v^2t^2 - vg\sin\theta\, t^3 + g^2t^4/4. \tag{51}$$

We want the derivative of ℓ (and thus ℓ^2) to never be less than zero. The derivative $d\ell^2/dt$ equals zero if

$$\begin{aligned} & 0 = 2v^2t - 3vg\sin\theta\, t^2 + g^2t^3 \\ \Longrightarrow\quad & 0 = g^2t^2 - 3vg\sin\theta\, t + 2v^2 \\ \Longrightarrow\quad & t = \frac{1}{2g^2}\left(3vg\sin\theta \pm \sqrt{9v^2g^2\sin^2\theta - 8v^2g^2}\right). \end{aligned} \tag{52}$$

A solution does *not* exist for t if the discriminant is less than zero, that is, if $\sin\theta < 2\sqrt{2}/3 \Longrightarrow \theta < 70.5^\circ$. So if θ is less than or equal to 70.5°, then ℓ never decreases during the flight.

3.57. **Rotating hoop**

The vertical component of the normal force must be mg, which implies that the horizontal component is $mg\tan\theta$. The horizontal $F = ma$ equation is then $mg\tan\theta = m(R\sin\theta)\omega^2 \Longrightarrow \omega = \sqrt{g/R\cos\theta}$. We see that the minimum ω occurs when $\theta = 0$, in which case it has the value $\sqrt{g/R}$. If ω is smaller than this, then the bead just sits at the bottom of the hoop.

4.16. **Angled rails**

Let x be the position of each mass along the rail, relative to the equilibrium position. Then the spring stretches a distance $2x\sin\theta$, yielding a force of $2kx\sin\theta$. The component of this force along the rail is $2kx\sin^2\theta$. So $F = ma$ along the rail gives $-2kx\sin^2\theta = m\ddot{x}$. Hence, $\omega = \sqrt{2k/m}\,\sin\theta$.