3.29. **Atwood's 3**

Define all accelerations positive upwards. By conservation of string, we have $a_2 = -(a_1 + a_3)/2$, because whatever mass disappears above m and $3m$ must appear above $2m$ and be divided evenly between the two segments there. The $F = ma$ equations are

$$T - mg = ma_1, \qquad 2T - 2mg = 2ma_2, \qquad T - 3mg = 3ma_3. \tag{22}$$

Solving these (the first two quickly give $a_1 = a_2$), along with $a_2 = -(a_1 + a_3)/2$, gives $a_1 = a_2 = g/5$, and $a_3 = -3g/5$.

3.39. **Equal distances**

We know from Eq. (3.38) that the horizontal distance is $2v_0^2 \sin\theta \cos\theta/g$. The time to the top is $v_0 \sin\theta/g$, so the maximum height is (looking at the ball fall back down to the ground) $gt^2/2 = v_0^2 \sin^2\theta/2g$. Equating these results gives $\tan\theta = 4$, so $\theta \approx 76°$.

3.62. **Radius of curvature**

(a) We have $a = v^2/r \implies r = v^2/a$. At the top, $v = v_0 \cos\theta$ and $a = g$. Therefore, $r = (v_0 \cos\theta)^2/g$.

(c) The maximum height is the usual $(v_0 \sin\theta)^2/2g$. So we want

$$\frac{v_0^2 \cos^2\theta}{g} = \frac{1}{2}\left(\frac{v_0^2 \sin^2\theta}{2g}\right) \implies \tan\theta = 2 \implies \theta \approx 63.4°.$$

3.70. **Stopping on a cone**

The $F = ma$ equation perpendicular to the surface of the cone gives

$$mg\sin\theta - N = (mv^2/R)\cos\theta \implies N = mg\sin\theta - (mv^2/R)\cos\theta. \tag{80}$$

The $F = ma$ equation along the direction of the motion gives $-\mu N = m(dv/dt)$, which yields

$$-\mu\, dt = \frac{dv}{g\sin\theta - (v^2/R)\cos\theta} \implies -\mu g\sin\theta \int_0^t dt = \int_{v_0}^0 \frac{dv}{1 - \frac{v^2}{gR\tan\theta}}. \tag{81}$$

Letting $u \equiv v/\sqrt{gR\tan\theta}$ gives

$$\begin{aligned} -\mu g\sin\theta \int_0^t dt &= \int_{v_0/\sqrt{gR\tan\theta}}^0 \frac{\sqrt{gR\tan\theta}\, du}{1-u^2} \\ \implies t &= -\frac{1}{2\mu}\sqrt{\frac{R}{g\sin\theta\cos\theta}}\ \ln\left(\frac{1+u}{1-u}\right)\Bigg|_{v_0/\sqrt{gR\tan\theta}}^0 \\ &= \frac{1}{2\mu}\sqrt{\frac{R}{g\sin\theta\cos\theta}}\ \ln\left(\frac{\sqrt{gR\tan\theta}+v_0}{\sqrt{gR\tan\theta}-v_0}\right). \end{aligned} \tag{82}$$

Note that if $v_0 = \sqrt{gR\tan\theta}$, then $t = \infty$. This makes sense, because this is the speed for which the string naturally makes an angle of θ with the vertical (as you can show); so the normal force is initially (and hence always) equal to zero. Also, if $\theta \to \pi/2$ (more precisely, if $v_0/\sqrt{gR\tan\theta} \ll 1$), then to lowest order the argument of the log is $1 + 2v_0/\sqrt{gR\tan\theta}$. So the log is essentially equal to $2v_0/\sqrt{gR\tan\theta}$. We then obtain $t \approx v_0/(\mu g\sin\theta) \approx v_0/(\mu g)$, which makes sense because the acceleration on flat ground is simply $a = -\mu g$. (Or more generally for $\theta \neq \pi/2$, if v_0 is very small, the normal force is essentially equal to $mg\sin\theta$, so the acceleration is $a = -\mu g\sin\theta$.)