2.20. **Block under an overhang**

Let's break up the forces into components parallel and perpendicular to the overhang. Let positive F_f point up along the overhang. Balancing the forces parallel and perpendicular to the overhang gives, respectively,

$$\begin{aligned} F_\mathrm{f} &= Mg\sin\beta + Mg\cos\beta, \qquad \text{and} \\ N &= Mg\sin\beta - Mg\cos\beta. \end{aligned} \tag{5}$$

N must be positive, so we immediately see that β must be at least 45° if there is any chance that the setup is static.

The coefficient μ tells us that $|F_\mathrm{f}| \le \mu N$. Using Eq. (5), this inequality becomes

$$Mg(\sin\beta + \cos\beta) \le \mu Mg(\sin\beta - \cos\beta) \quad\Longrightarrow\quad \frac{\mu+1}{\mu-1} \le \tan\beta. \tag{6}$$

We see that we must have $\mu > 1$ in order for there to exist any values of β that satisfy this inequality. If $\mu \to \infty$, then β can be as small as 45°, but it can't be any smaller.

2.21. **Pulling a block**

The F_y forces tell us that $N + F\sin\theta - mg = 0 \Longrightarrow N = mg - F\sin\theta$. And assuming that the block slips, the F_x forces tell us that $F\cos\theta > \mu N$. Therefore,

$$F\cos\theta > \mu(mg - F\sin\theta) \quad\Longrightarrow\quad F > \frac{\mu mg}{\cos\theta + \mu\sin\theta}\,. \tag{7}$$

Taking the derivative to minimize this then gives $\tan\theta = \mu$. Plugging this θ back into F gives $F > \mu mg/\sqrt{1+\mu^2}$. If $\mu = 0$, we have $\theta = 0$ and $F > 0$. If $\mu \to \infty$, we have $\theta \approx 90^\circ$ and $F > mg$.

2.22. **Holding a cone**

Let F be the friction force at each finger. Then the F_y forces on the cone tell us that $2F\cos\theta - 2N\sin\theta - mg = 0$. But $F \le \mu N$. Therefore,

$$2\mu N\cos\theta - 2N\sin\theta - mg > 0 \quad\Longrightarrow\quad N \ge \frac{mg}{2(\mu\cos\theta - \sin\theta)}\,. \tag{8}$$

This is the desired minimum normal force. When $\mu = \tan\theta$, we have $N = \infty$. So $\mu = \tan\theta$ is the minimum allowable value of μ.

2.31. **Cylinder and hanging mass**

If T is the tension in the string, then $T = mg$. If F is the friction force from the plane, then balancing torques around the center of the cylinder gives $F = T$, so F also equals mg. If N is the normal force from the plane, then balancing horizontal forces on the cylinder gives $N \sin\theta = F\cos\theta \implies N = mg/\tan\theta$. Finally, balancing vertical forces on the cylinder gives

$$\begin{aligned} N\cos\theta + F\sin\theta - Mg - T = 0 \quad &\implies \quad \left(\frac{mg}{\tan\theta}\right)\cos\theta + (mg)\sin\theta - mg = Mg \\ &\implies \quad m = \left(\frac{\sin\theta}{1-\sin\theta}\right)M. \end{aligned} \tag{12}$$

If $\theta = 0$, then $m = 0$. And if $\theta \to 90^\circ$, then $m \to \infty$. These make sense.

Alternatively, once we know that $T = mg$, we can just use torque around the contact point on the plane, which doesn't require knowing F or N. The lever arm for the Mg force is $R\sin\theta$, and the lever arm for the T force is $R(1-\sin\theta)$. Balancing the torques around the contact point therefore gives $(Mg)R\sin\theta = (mg)R(1-\sin\theta)$, in agreement with the above result.

2.35. **Two sticks and a string**

(a) Balancing vertical forces on the whole system tells us that the normal forces at the bottoms of the sticks must sum to $2mg$. Balancing torques on the whole system around the hinge then tells us that these normal forces must be equal, and hence both equal to mg. Finally, balancing torques on the right stick around the hinge tells us that the tension T in the string satisfies

$$T(\ell\cos 2\theta) + mg(\ell/2)\sin\theta = mg\ell\sin\theta \quad \implies \quad T = \frac{mg\sin\theta}{2\cos 2\theta}\,. \tag{14}$$

(b) Look at the forces on the right stick. The mg forces (gravity and normal force) cancel. Therefore, the force from the hinge must cancel the tension. So the hinge force points up to the right (perpendicular to the stick) with magnitude $mg\sin\theta/(2\cos 2\theta)$.