Name: ____________________

Date: ____________________

Collaborators: ____________________

(Collaborators submit their individually written assignments together, in class, in person)

Homework rules:

Show your work and indicate your reasoning in order to receive credit.

Guessing answers is not a solution.

You can use calculators, Computer Algebra systems, or any programming in general **only if the problem statement requests that**.

To solve the problems, use the methods that are taught in this class. If a problem statement includes directions for solutions, following those directions is required.

Question:	1	2	3	Total
Points:	25	25	25	75
Score:				

Instructor/grader comments:

The method of residues

1. (25 points) Calculate the following integral for real a, $|a| < 1$, and integer n, $n \geq 0$:

$$I_n(a) = \int_{-\pi}^{\pi} \frac{\sin(n\varphi)}{1 - 2a\sin(\varphi) + a^2}\, \mathrm{d}\varphi.$$

Sketch the integration contour in the complex plane. Indicate the position(s) of the pole(s) of the integrand.

To verify your solution, use a computer algebra system to plot a graph of $I_n(a)$ for $0.1 \leq a \leq 0.8$ and $n = 3$ and $n = 5$. The expected graph is shown in Fig. 1.

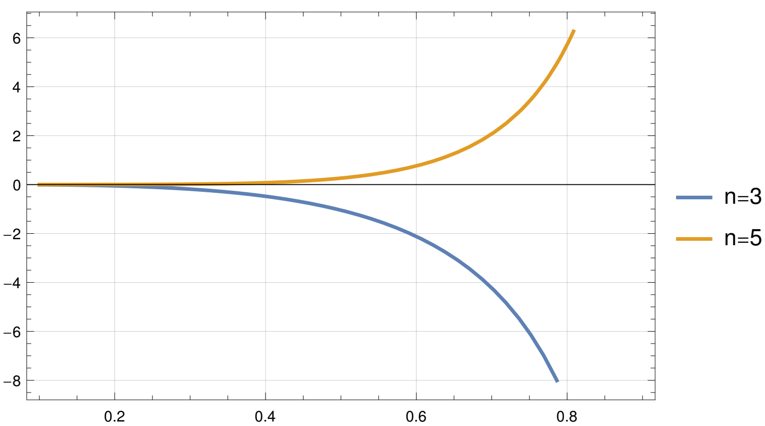

Figure 1: Expected graph in Problem 1 (colors online)

Attach a printout of your CAS session.

Hint: Consider the integral

$$J_n(a) = \int_{-\pi}^{\pi} \frac{e^{in\varphi}}{1 - 2a\sin(\varphi) + a^2}\, \mathrm{d}\varphi.$$

2. (25 points) Calculate the integral for $a > 0$:

$$I(a) = \int_0^{\infty} \frac{x\, \mathrm{d}x}{a + x^3}.$$

Use the integration contour shown in Fig. 2 where $R \to \infty$. Sketch the position(s) of the pole(s) of the integrand.

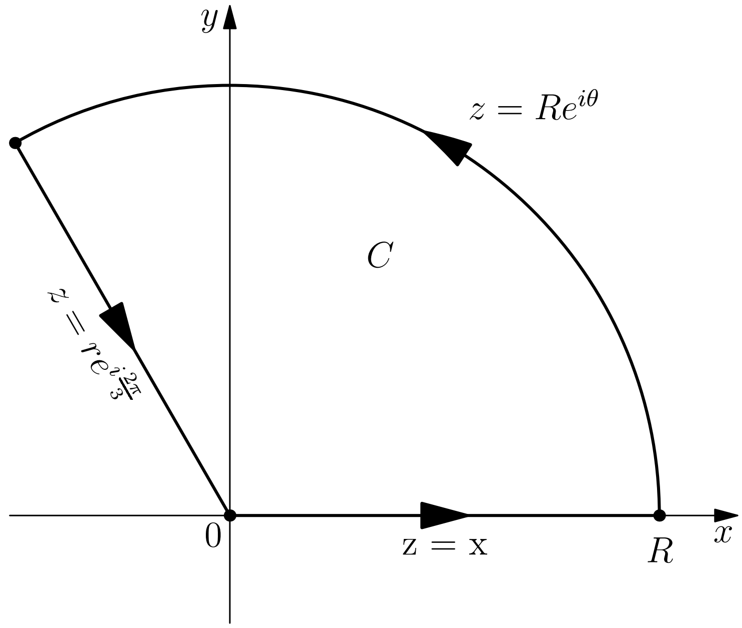

Figure 2: Integration contour for Problem 2.

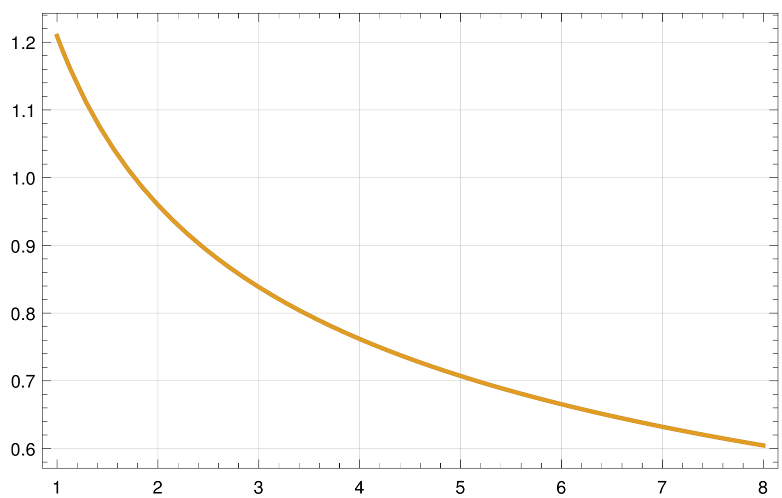

Figure 3: Expected graph in Problem 2

To verify your solution, use a computer algebra system to plot a graph of $I(a)$ for $1 \le a \le 8$. The expected graph is shown in Fig. 3.

Attach a printout of your CAS session.

3. (25 points) Calculate the integral (assume $a > 0$):

$$I(a) = \int_0^\infty \frac{\mathrm{d}x}{\left(a^2 + x^2\right)^2}.$$

Sketch the integration contour. Indicate the position(s) of the pole(s) of the integrand.

To verify your solution, use a computer algebra system to plot a graph of $I(a)$ for $3 \leq a \leq 8$. The expected graph is shown in Fig. 4.

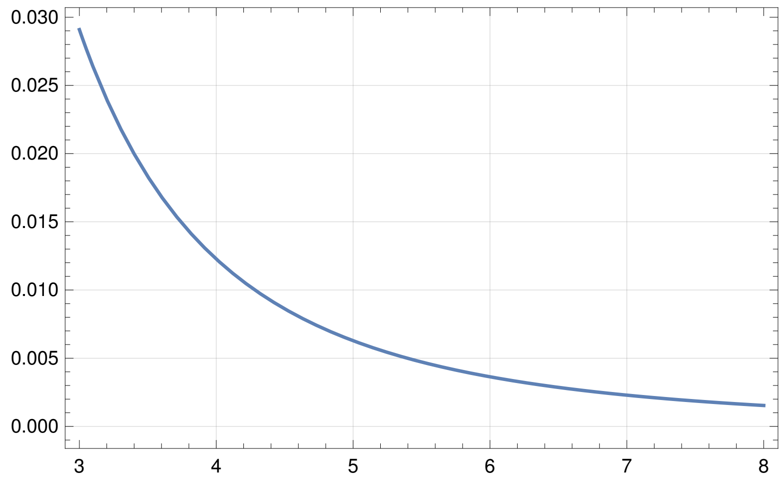

Figure 4: Expected graph in Problem 3

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