Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together and in person)

Homework rules:

Show your work and indicate your reasoning in order to receive credit.

Guessing answers is not a solution.

You can use calculators, Computer Algebra systems, or any programming in general **only if the problem statement requests that.**

To solve the problems, use the methods that are taught in this class. If a problem statement includes directions for solutions, following those directions is required.

Question:	1	2	3	4	Total
Points:	10	20	25	15	70
Score:					

Instructor/grader comments:

Cauchy-Riemann equations

1. (10 points) Verify that that the following function

$$u(x, y) = x^2 - 3y^2$$

cannot be a real part of any complex function $f(z)$.

In this problem, do not attempt to integrate Cauchy-Riemann equations.

2. (20 points) Use Cauchy-Riemann equations to find the analytic function $f(z)$, $z = x + iy$, such that its real part is as following:

$$\text{Re}\, f(z) = u(x, y) = e^x \sin y,$$

and

$$f(i\pi) = 0.$$

Express the result for $f(z)$ as a **function of z only.**

Answer: $f(z) = -i\,(e^z + 1)$.

The Cauchy integral theorem

3. (25 points) Evaluate the integral in terms of Gamma function:

$$I = \int_0^\infty \sin\left(x^3\right) \mathrm{d}x$$

Answer: $I = \frac{1}{2}\Gamma\left(\frac{4}{3}\right)$.

Hints: consider the integral

$$\oint_C e^{-z^3}\, \mathrm{d}z$$

along the contour C sketched in Fig. 1; use the Euler formula; use the relation

$$\int_0^\infty e^{-x^3}\, dx \equiv \Gamma\left(\frac{4}{3}\right),$$

where Γ is gamma function.

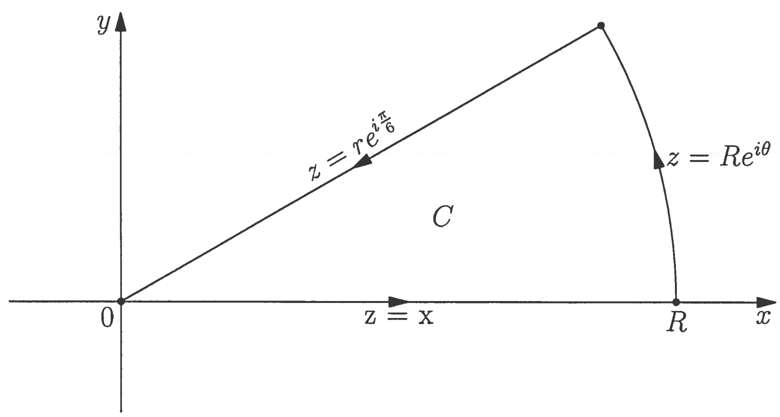

Figure 1: Integration contour for Problem 3. ($R \to \infty$) .

Accept without a proof that the integral over the circular arc in Fig. 1 goes to 0 as $R \to \infty$.

Cauchy integral formula

4. (15 points) Evaluate the integral

$$I = \oint_C \frac{e^{-iz}}{z^2(z-\pi)}\,\mathrm{d}z,$$

where C is a closed contour in the complex plane z that includes the point $z = \pi$ and does not include the origin (see Fig. 2).

Answer: $I = -\frac{2i}{\pi}$.

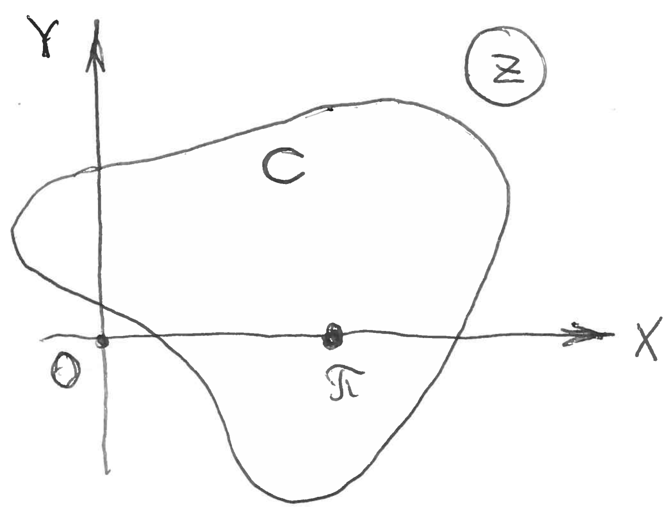

Figure 2: Integration contour for Problem 4.