Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together, in class, in person)

Question:	1	2	3	4	Total
Points:	15	15	15	25	70
Score:					

Instructor/grader comments:

Computer algebra

1. (15 points) Consider the following integral:

$$I(x) = \int_{\frac{1}{2}}^{\pi-\frac{1}{2}} e^{-(y-\pi/2)^6} \sin^x(y)\, \mathrm{d}y\,. \tag{1}$$

   a. Verify that Mathematica cannot obtain an analytic expression for the integral Eq. (1). Use Mathematica function `Integrate[ ]`.

   b. Define the function, `f1[x_]`, that evaluate the integral numerically. Use Mathematica function `NIntegrate[ ]`.

   c. Plot on the same graph, for $10 \le x \le 50$, your function and the following approximation to the integral:

$$g1(x) = \sqrt{\frac{2\pi}{x}}.$$

   Provide a grid and a legend. The resulting graph should look similar to Figure 1.

   (We are going to learn how to obtain approximations for this and similar integrals later in the course.)

   d. Print your Mathematica session and attach the printout to the rest of your homework.

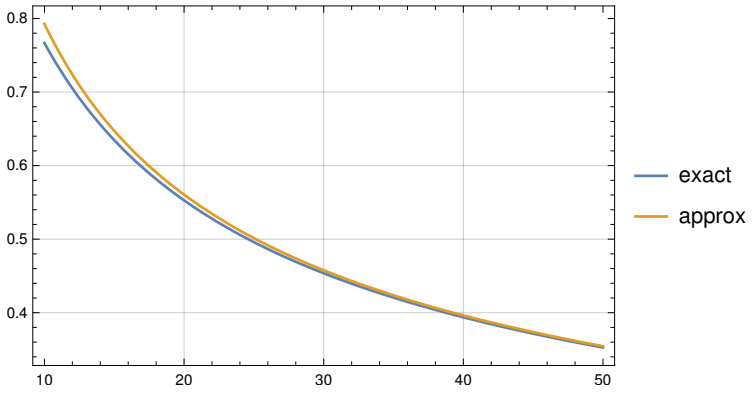

Figure 1: Expected graph in Problem 1 (color online).

2. (15 points) Consider the following two-point boundary value problem:

$$\epsilon\, y'' + 2y' + e^y = 0, \quad y(0) = 0, \quad y(1) = 0, \quad \epsilon = 1/20. \tag{2}$$

   a. Verify that Mathematica cannot obtain an analytic solution for the problem Eq. (2). Use Mathematica function `DSolve[]`.

   b. Solve the boundary value problem numerically. Use Mathematica function `NDSolve[]`.

   c. Plot on the same graph, for $0 \le x \le 1$, the numerical solution of the boundary value problem and the following approximation to the solution:

$$g2(x) = \log\left(\frac{2}{1+x}\right) - \log(2)\exp\left(-\frac{2x}{\epsilon}\right)$$

   Provide a grid and a legend. The resulting graph should look similar to Figure 2.

   (We are going to learn how to obtain approximations for solutions of differential equations later in the course.)

   d. Print your Mathematica session and attach the printout to the rest of your homework.

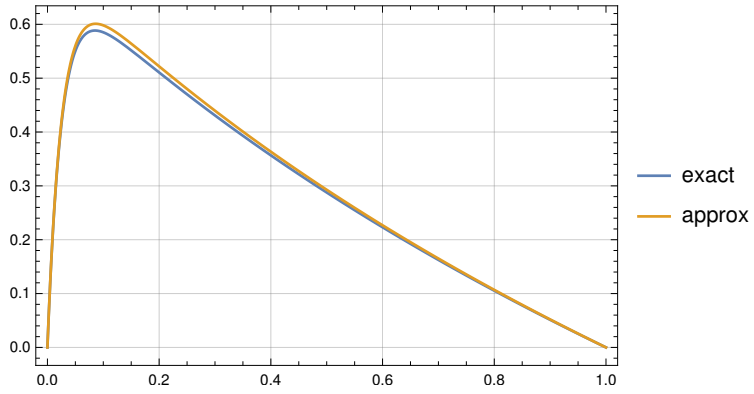

Figure 2: Expected graph in Problem 2 (color online).

3. (15 points) Consider the following expression:

$$\ln(n!),$$

where $n! \equiv 1 \times 2 \times 3 \cdots (n-1) \times n$ is the factorial of integer n.

   a. Define the function, `f3[x_]`, that evaluate the above expression. Use Mathematica function `Factorial[]`.

   b. Plot on the same graph, for $1 \leq x \leq 5$, your function and the following approximation:

$$g3(x) = \log(2\pi)/2 + (1/2 + x) * \log(x) - x$$

   Provide a grid and a legend. The resulting graph should look similar to Figure 3.

   (We are going to learn how to obtain approximations for sums and products later in the course.)

   c. Print your Mathematica session and attach the printout to the rest of your homework.

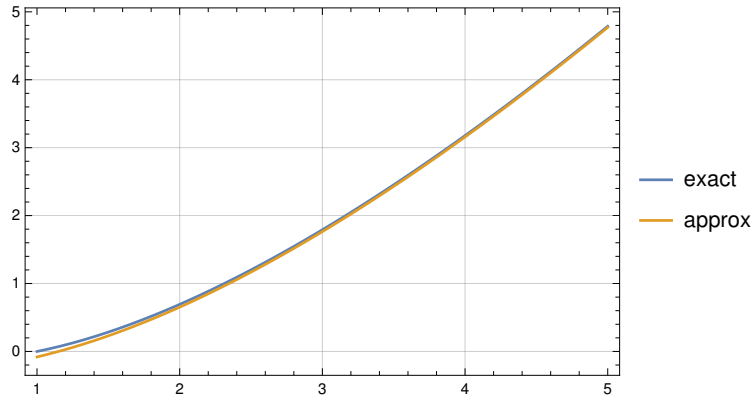

Figure 3: Expected graph in Problem 3 (color online).

Course concepts

4. (a) (15 points)

   ☐ I've watched in full the video recording of R. Feynman's lecture *The relation of Mathematics and Physics*.

   (b) (10 points)

   ☐ I've read the Introduction, pp. 9–13, to the lecture notes *Physical Mathematics*, by Michael P. Brenner.

Sign and date here: _____