Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	4	5	Total
Points:	20	15	15	20	15	85
Score:						

Instructor/grader comments:

Method of stationary phase

1. (a) (15 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 \cos(\lambda x^p)\, \mathrm{d}x$$

for p real and $p > 1$.

Describe the location of the stationary point of the integrand, and the approximation you used for the integrand in its vicinity.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $p = 3$ and $10 \le \lambda \le 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 1.

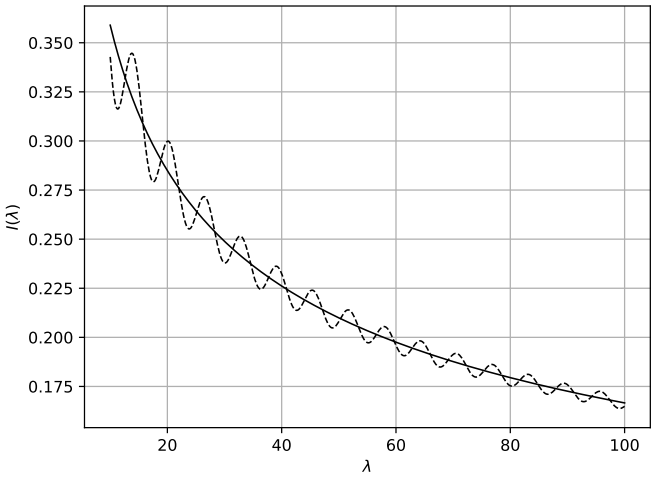

Figure 1: Expected result in Problem 1 (solid line – asymptotics, dashed line – numerically evaluated integral).

2. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\infty \cos\left(\lambda x^2 - x\right) \mathrm{d}x.$$

Describe the location of the stationary point of the integrand, and the approximation you used for the integrand in its vicinity.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $16 \le \lambda \le 1024$. Use logarithmic x-axis. Attach a printout of your CAS session.

The expected graph is shown in Fig. 2.

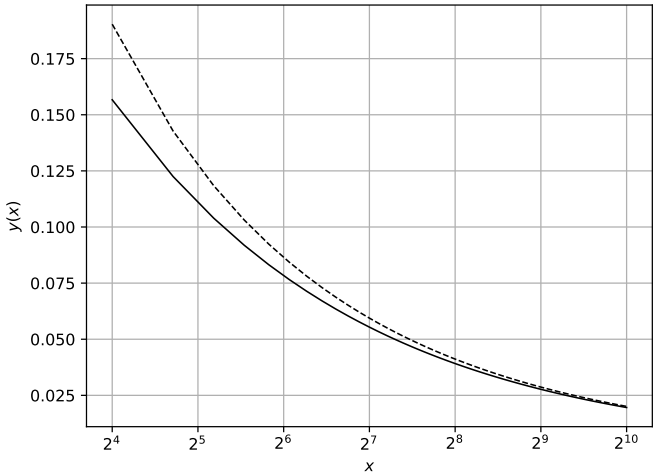

Figure 2: Expected result in Problem 2 (solid line – asymptotics, dashed line – numerically evaluated integral).

3. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\pi \cos\left(\lambda \cos(x)\right) \mathrm{d}x.$$

Describe the location of the stationary point of the integrand, and the approximation you used for the integrand in its vicinity.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $1 \le \lambda \le 30$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 3.

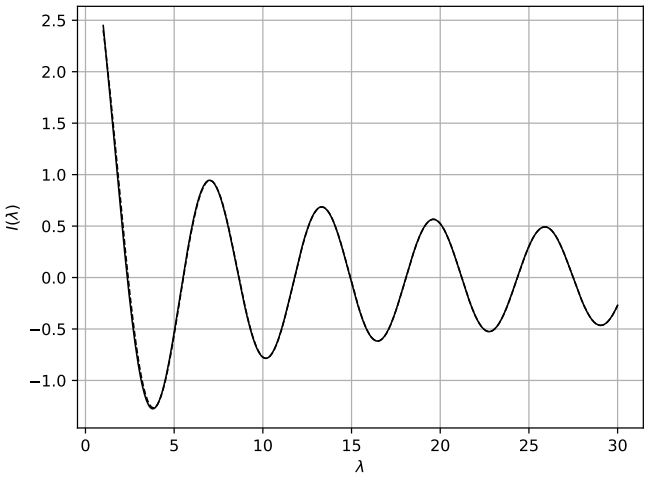

Figure 3: Expected result in Problem 3 (solid line – asymptotics, dashed line – numerically evaluated integral).

Integration by parts

4. (a) (15 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_1^\infty \cos\left(\lambda x^2\right) \mathrm{d}x.$$

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $10 \le \lambda \le 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 4.

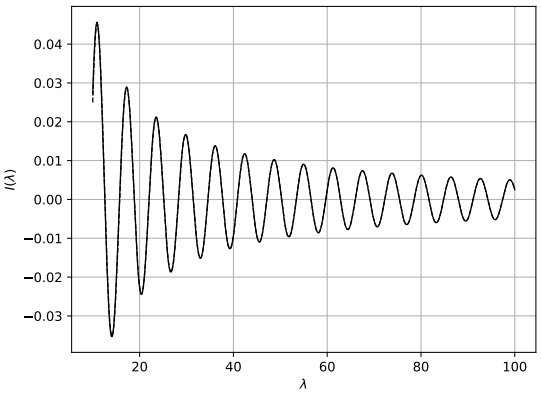

Figure 4: Expected result in Problem 4 (solid line – asymptotics, dashed line – numerically evaluated integral).

5. (a) (10 points) Improve the approximation for the integral you considered in Problem 1 by evaluating the first correction term for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 \cos\left(\lambda x^3\right) \mathrm{d}x.$$

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $10 \le \lambda \le 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 5.

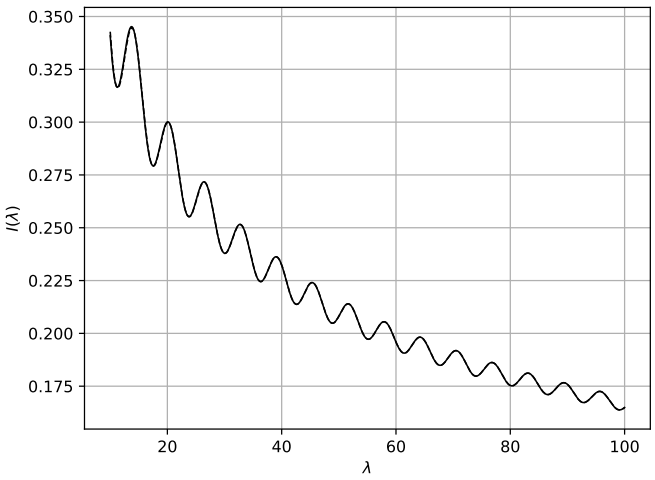

Figure 5: Expected result in Problem 5 (solid line – asymptotics, dashed line – numerically evaluated integral).