Name: _____

Date: _____

Collaborators: _____

(Collaborators submit their individually written assignments together and in person)

Question:	1	2	3	4	Total
Points:	25	15	30	10	80
Score:					

Instructor/grader comments:

Laplace method for integrals

1. (a) (20 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\pi \frac{\mathrm{d}x}{(x^3+1)^\lambda}.$$

Clearly describe the location of the maximum of the integrand, and the approximation you used for the integrand in the vicinity of the maximum.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $10 < \lambda < 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 1.

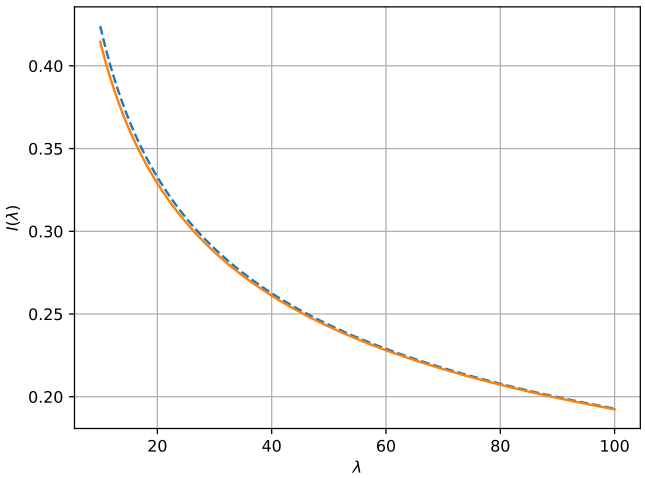

Figure 1: Expected result in Problem 1.

2. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{-3}^{2} e^{-\lambda \sinh^2 x} \, \mathrm{d}x.$$

Clearly describe the location of the maximum of the integrand, and the approximation you used for the integrand in the vicinity of the maximum.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $5 < \lambda < 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 2.

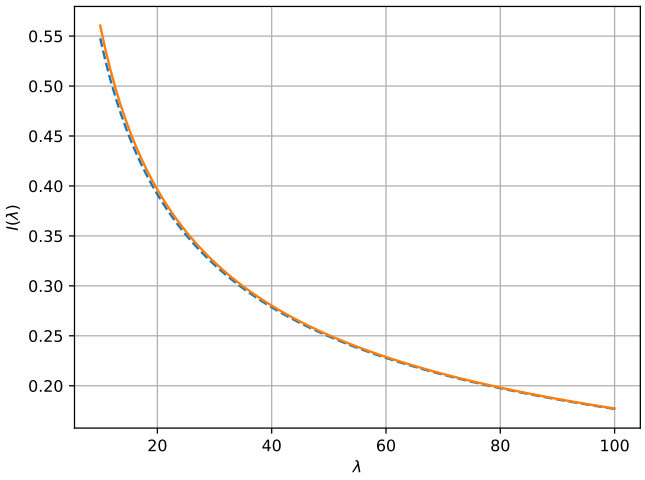

Figure 2: Expected result in Problem 2.

3. (a) (25 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\infty e^{-\lambda x - \frac{4}{x^2}} \mathrm{d}x.$$

Hint: the integrand in this problem has a *moving maximum*. You need to change the integration variable so that the maximum of the integrand occurs at a fixed point independent on λ.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $3 < \lambda < 7$. Use log scale for y axis. Attach a printout of your CAS session.

The expected graph is shown in Fig. 3.

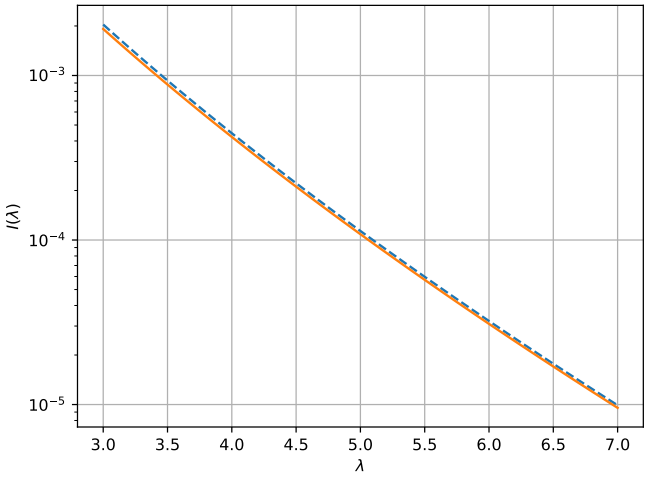

Figure 3: Expected result in Problem 3.

The method of dominant balance

4. (10 points) For $0 < \epsilon \ll 1$ find the leading in power of ϵ term for the roots of the following polynomial equation:

$$\epsilon x^5 - (x-1)^3 = 0.$$

Hints/directions:

- The polynomial is of the fifth degree in x, therefore it has 5 roots.
- It is easy to obtain the leading in ϵ term of the triple root of the equation, $x = 1$, (counted as three roots), if we neglect the therm ϵx^5. Note that the balance is consistent since we dropped the term $\epsilon x^5 = \epsilon$ but kept four terms x^3, $3x^2$, $3x$, and 1, each of order one.
- To find the remaining two roots, we must take into account the term ϵx^5 and balance it with one of the remaining four terms.
- To simplify your calculations,
  - choose the balancing term such that you get an equation with two roots. Verify that your balance choice is consistent.
  - once you found the consistent choice of balance that gives you two roots, do not consider the remaining balancing options.