Name: ______________________________

Date: ______________________________

Collaborators: ____

Question:	1	2	3	Total
Points:	25	25	25	75
Score:				

Instructor/grader comments:

Integral to stump a computer algebra system

1. (a) (20 points) Construct a definite integral that you can evaluate analytically but a computer algebra cannot. Use the method described in the handout "The integral that stumped Feynman". Do not use the integrands similar to ones discussed in the handout. Use a computer algebra system for finding the real and the imaginary parts of your complex expressions.

   (b) (5 points) To verify your result, numerically evaluate your integral and your answer. Use a computer algebra system for numerics. Enclose a printout of you computer algebra session.

The method of residues

2. (25 points) Calculate the following integral for $a > 0$:

$$I(a) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan(x + a\,i)\,\mathrm{d}x.$$

Sketch the integration contour. Indicate the position(s) of the pole(s) of the integrand. To verify your answer, consider the limit $a \to \infty$. (Note that $I(\infty) = i\,\pi$; extra credit for showing this without using the method of residues.)

Hints: note that the integrand is a periodic trigonometric function and the integral is over its period. However, the 'standard' change of variables $z = e^{ix}$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ does not result in a closed contour in a complex z-plane.

3. (25 points) Calculate the following integral for $a, b > 0$:

$$I(a,b) = \int_0^{\infty} \frac{x^4\,\mathrm{d}x}{(bx^2 + a)^4}.$$

Answer: $I(a,b) = \pi/\left(32\,a^{\frac{3}{2}}\,b^{\frac{5}{2}}\right)$

Hints: recall that for a pole of order n, $n > 1$, at $z = z_0$ the residue is

$$\mathrm{Res}\,(f(z), z = z_0) = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{\mathrm{d}^{(n-1)}}{\mathrm{d}z^{(n-1)}} \left[f(z)\,(z - z_0)^n\right].$$

Use a computer algebra system to calculate the derivatives in the expression for the residue and simplify the expressions. Use the formula above; do not use any additional tools that you computer algebra system may provide.